



Financial Crisis: Origins, Macroeconomic Consequences and Policy Response

Paweł Kopiec

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Florence, 12 September 2016

European University Institute
Department of Economics

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Dedykuje mojej Żonie / To my wife

Acknowledgements

My deepest gratitude goes to my advisor, professor Arpad Abraham for his valuable guidance, constant encouragement and hundreds of hours spent discussing my research. I would like to thank professor Piero Gottardi for his well-aimed criticism and attention to details that helped me to structure my ideas at all stages of my work. I am grateful to professor Wouter den Haan for his invaluable comments and suggestions on my research during my visit at LSE. My stay in London was very inspiring and gave me lots of motivation.

Profound gratitude goes to my dear friends: Krzysztof Pytka and Pawel Doligalski for their insights and stimulating discussions. I would like to thank the administrative staff at the EUI: Lucia Vigna and Jessica Spataro for their constant readiness to help in dealing with all official matters.

Special thanks are given to my wife for her unconditional love. The support of my family was fundamental - I owe all to you.

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Abstract

The global financial crisis of 2007-2008 is considered to have been the worst economic recession since the Great Depression. Its beginning is associated with the bursting of the US housing bubble in 2007 and the financial panic of 2008. It led to a collapse of many financial institutions and others were prevented from bankruptcy by the bailouts provided by national governments. Malfunctioning debt markets and increased uncertainty played a crucial role in transmission of the financial disturbances to the real sector. This in turn caused large drops in output and dramatic hikes in unemployment rates across the developed countries that persisted for a long period of time after the onset of the financial crisis. Economic slowdown triggered an unprecedented response of central banks (through balance sheet expansions) and governments (through fiscal stimuli). In this thesis I address topics that are associated with three subsequent stages of the crisis.

In Chapter 1, I analyze a question that concerns the origins of the financial collapse. More precisely, I study the impact of changes in competition in the banking industry on financial stability and business cycle dynamics. This paper is motivated by a heated debate that started in aftermath of the crisis: many economists pointed out that the financial deregulation of the 1970s and the 1980s was one of the main causes of the global crisis of 2007-2008. Chapter 2 is related to economic phenomenon that is characteristic for the stage that follows the financial turmoil - high unemployment. In particular, I construct unemployment decomposition method based on the DSGE model that enables to divide the observed rate of unemployment into frictional and Keynesian components. I use this procedure to analyze the unemployment structure in four European economies: Germany, France, Italy and Spain. The last part of my thesis - Chapter 3 - is devoted to the stage of crisis in which government takes actions that are aimed at fighting the negative macroeconomic consequences of financial collapse. More specifically, I build a tractable framework with search frictions in the market for products and simple supply structure within the manufacturing sector to discuss the impact of an increase in government spending on aggregate output and consumption.

1 Competition in the Financial Sector and Financial Crises in a Business Cycle Model

Abstract

In this theoretical work, I study a dynamic general equilibrium model with financial sector in which aggregate activity depends on the conditions of intermediaries' balance sheets. This environment is used to demonstrate the business cycle consequences of changes in competition in the financial industry. On the one hand competitive banking sector is associated with higher average level of aggregate output. On the other hand, however, monopolistic financial industry increases financial and macroeconomic stability. This trade-off is present both in the short-run and in the long-run. Additionally, the impact of changes in aggregate risk on performance of various market structures is studied. Despite the model's dynamic structure and agents' heterogeneity the results presented in this paper are analytical.

1.1 Introduction

The goal of this paper is to investigate a controversial questions that have arisen during the recent financial crisis: is financial stability enhanced or weakened by the competition in the financial industry? What are the business cycle implications of changes in the financial sector's market structure?

A significant increase in competition in the US financial sector started in early 1970's when many nondepository financial institutions began to offer financial services that were closely related to those offered by standard firms (e.g., commercial banks, savings and loan associations, credit unions).¹ Activities of the latter, however, were heavily regulated which decreased their ability to compete with the new financial products offered by nondepository firms. For instance, brokerage firms started to offer credit for real estate and related purposes or Cash Management Accounts (CMA) - services that were directly competitive with those offered by depository financial institutions. Another example of financial innovations provided by nondepository firms that resembled the services of depository institutions were those offered by money market funds. They originated in 1970's and offered savers a market rate of interest at a time when the rates available at traditional depository institutions were constrained by the so called Q ceilings. Money market funds invested their funds in short-term, high-quality money market instruments as T-bills or commercial paper. The process of financial innovation made depository institutions (that were heavily regulated) press for change in the financial system and seek legislative changes that would allow them to compete with nondepository institutions. These actions were strengthened by pressures of consumer groups on Congress. For instance, the elderly argued that Q ceilings discriminated small savers and postulated elimination of those constraints on depository institutions. Finally, in 1980 and 1982 two reform legislations that deregulated the financial system

¹A comprehensive description of this process can be found in [Cooper and Fraser (1986)].

were signed by President Carter and President Reagan. The first one, the Depository Institutions Deregulation and Monetary Control Act of 1980 began the process of elimination of Regulation Q interest rate ceilings on deposit accounts at depository institutions. Moreover, it authorized all depository institutions to offer interest-bearing transaction accounts which broke the traditional monopoly of commercial banks on these instruments. Additionally, savings and loans were allowed to commit a significant fraction of their assets to consumer loans. The second reform - the Garn - St Germain Depository Institutions Act of 1982 gave further powers to savings and loans: they were permitted to offer demand deposits to commercial, agricultural and corporate customers and to expand their lending activities. On the top of that, the 1982 legislation allowed the depository institutions to create money market deposit accounts so that they are able to compete with money market funds. The deregulation introduced by those two acts increased the competition in the financial sector in a significant way. Conventional wisdom links the deregulation of financial markets that started in 1980's with financial instability and the financial crisis of 2007 and 2008: *"The severity of today's financial crisis is blamed by some on the pressure of competition on banks. (...) the lifting of restraints, such as interest-rate caps on deposits or rules that prevent banks from operating in certain markets, leads to more intense competition. That is good for borrowers, but it also hurts banks' profit margins"*.² Lower margins led to lower profits and made it harder for banks to collect equity. This in turn resulted in financial instability and was one of the causes of the recent financial crisis.

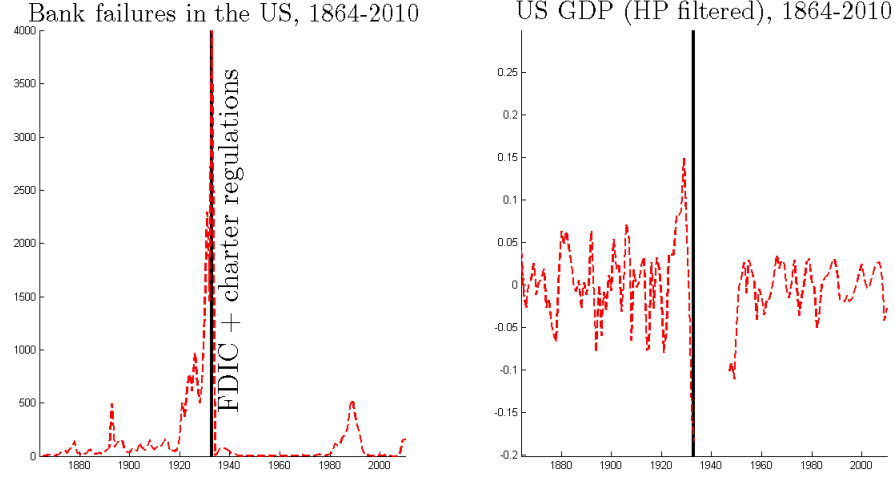
A longer time perspective that concerns the relationship between business cycle fluctuations and competition in the financial industry was presented by [Gorton (2010)]. As he points out: *"The period from 1934, (...), until the current crisis is somewhat special in that there were no systemic banking crises in the United States"*. According to Gorton, one reason for which the banking system in the United States was panic-free between 1934-2007 was: *"In addition to bank regulation, bank charters were valuable because of subsidies in the form of limited entry into banking, local deposit monopolies, interest rate ceilings (...). But the value of a bank charter eroded in the 1990s with increased competition from nonbanks"*. Gorton adds that: *"The period of quiescence is related to what macroeconomists call 'The Great Moderation', a view associated with the observation that the volatility of aggregate economic activity has fallen dramatically in most of the industrialized world. One explanation for this is that there were no longer banking panics"*. Figure 1 presents the relationship between the number of bank failures/suspensions and GDP per capita in the US between 1864 and 2010.³

Motivated by these examples, I study the impact of changes in competition in the banking sector on financial stability and business cycle performance of economy. To conduct my analysis I construct a tractable business cycle model with a single source of aggregate uncertainty - shocks to preferences of capital buyers. It has several features that make it useful from the point of view of the goal of this paper. First, it includes financial sector and its role is to transfer funds from agents who do not have investment opportunities to those who have them. Second, the amount of aggregate investment

²"*Deliver us from competition*", *The Economist*, 25.06.2009

³To construct the series for bank suspensions/failures before 1933 I have used *Historical Statistics of the United States: colonial times to 1970* published by the US Department of Commerce and for period after 1934 I have used the FDIC data on *Failures and Assistance Transactions*. The data on GDP per capita are taken from the Angus Maddison's database.

Figure 1: The period of quiescence in banking and "The Great Moderation"



is associated with conditions of banks' balance sheets. In particular, if equity of financial institutions is drained by adverse aggregate shock then intermediation activities are impeded. This in turn means that less resources are transferred to investors and hence the level of aggregate investment drops. Third, intermediaries provide depositors with safe assets with return that does not depend on realization of aggregate shocks.⁴ Fourth, my specification allows for comparisons of market structures characterized by different intensities of the monopoly power of financial institutions. From technical point of view my model is related to two well-established frameworks in the literature. The first one is the model of [Kiyotaki and Moore (2012)] and the second one is [Bigio (2015b)].

Results are theoretical and can be divided into three subgroups. First of them contains analytic outcomes that describe the short-run trade-off between competitive and monopolistic banks.⁵ In particular, it focuses on the interplay of two opposite forces: on the one hand, competitive banks channel more funds to investors which leads to higher production of investment goods, increases accumulation of capital and boosts output. On the other hand, however, competitive bankers exhibit greater risk exposure and hence incur more severe losses during recessions which drain their equity and may impede intermediation activities in the future.

Second subgroup analyzes the long-run behavior of economies with two different financial sectors (competitive and monopolistic banks). More precisely, it concentrates on properties of ergodic distributions of aggregate state variables - aggregate capital and banks' equity under both financial regimes.⁶ As we shall see later, the level of capital pins down the level of aggregate output and the amount of banks' equity determines financial stability and vulnerability to financial disturbances. Through the

⁴This feature implies that financial intermediaries in my model are similar to standard banks as they provide agents with services that bear resemblance to deposits.

⁵This situation is dubbed "short-run" as both regimes - the one with monopolistically competitive banks and the one with competitive banks start with the same initial values of state variables.

⁶Capital and banks' equity are the only state variables in my model.

lens of the model, I evaluate the plausibility of the following hypotheses that describe the long-run interaction between capital and equity:

Hypothesis 1. Competitive banks provide entrepreneurs with more intermediation services and at the same time they earn lower margins per each unit of capital that is channeled by them (because of the absence of monopolistic wedge). The former factor raises aggregate investment, increases capital and output. This in turn boosts demand for intermediation services and the volume of capital transferred by banks grows. Extensive margin of intermediation services is greater which compensates lower intensive margin earned by competitive banks. As a result competitive banks generate higher profits and are able to accumulate more equity than monopolistic intermediaries. This means that competitive financial sector is more stable and guarantees better macroeconomic outcomes (higher aggregate output) than the monopolistic one.

Hypothesis 2. Monopolistic banks exercise their market power and generate higher profits. The latter enables them to build greater equity cushion. Since the amount of financial services is positively related to amount of banks' equity then banks channel more funds which increases investment and output. This force, as a consequence, outweighs the impact of the monopolistic distortion on the amount of intermediation services. This means that monopolistic financial sector guarantees both higher stability and higher output and hence is more beneficial than the competitive market structure.

Hypothesis 3. There is a trade-off in the long-run. Competitive banks provide entrepreneurs with larger amount of cheaper intermediation services but lower profit margins generated by them hinder accumulation of equity. This in turn deteriorates financial (and macroeconomic) stability which gives rise to trade-off that is similar to the one present in the short-run perspective.

My analysis rejects Hypotheses 1 and 2 and predicts that Hypothesis 3 is true.

Third subgroup of results concerns the impact of aggregate risk on the behavior of two regimes. In particular, I check how the magnitude of "bad" shocks affects the long-run trade-off discussed above. It may appear that competitive financial sector is significantly outperformed (in terms of financial and macroeconomic stability) by the monopolistic sector as financial disturbances become more severe (this may occur because competitive banks are not able to accumulate sufficient equity cushion to buffer adverse aggregate shocks). This intuition ignores the response of competitive bankers to changes in aggregate environment: since they predict the devastating impact of deeper "bad" shocks on their balance sheets, their behavior becomes more precautionary. More precisely, they decide to channel less resources to entrepreneurs that hold investment opportunities and as a result ergodic distribution of banks equity under competitive banks converges to the one that characterizes economy with monopolistic regime.

The rest of the paper is organized as follows. In Section 1.2 I discuss the literature that is related to my analysis and I present contributions of this work. In Section 1.3 the business cycle model with perfectly competitive banks is shown and the transmission mechanism of aggregate shocks is presented. Section 1.4 describes the model with monopolistically competitive intermediaries - it is formulated in such a way that the model with perfectly competitive banks is a special subcase of this construction. In Section 1.5 an analytic comparison of two regimes: economy with perfectly competitive financial institutions and economy with monopolistically competitive banks is made and two types of trade-offs

(that emerge in the short-run and in the long-run) are presented. Sections 1.6 and 1.7 study the impact of exogenous changes in aggregate risk on both regimes and sources of inefficiency of allocation in economy with competitive banks, respectively. Section 1.8 concludes.

1.2 Literature

The paper is related to several strands in the literature.

Market structure and financial stability. The first, theoretical strand, concerns the effects of changes in banking sector's market structure on stability of the banking sector. There are two main approaches within this literature: the risk-shifting view and the charter value view. The risk-shifting theory, represented by the article of [Boyd and De Nicro (2005)] (that builds on the seminal work of [Stiglitz and Weiss (1981)]), assumes that higher interest rates (on bank loans), that are associated with an increase in the monopoly power of the banking sector, will make firms invest in riskier projects which in turn translates into higher banks' portfolio risk and gives rise to financial instability. The charter value hypothesis, originated with the article by [Keeley (1990)], postulates that a decrease in competition in the banking industry increases banks' future profits generated by the market power. This in turn makes banks more cautious when making their investment decisions, since bankruptcy means that they lose the valuable stream of future rents. [Martinez-Miera and Repullo (2010)] try to reconcile the two aforementioned views. They claim that on the one hand when (as a result of decrease in intermediaries' monopoly power) banks charge lower rates, their borrowers choose safer investments, so their portfolios are safer (like in [Boyd and De Nicro (2005)]). On the other hand, lower interest rates on loans decrease banks' profits which serve as a buffer against loan losses. Those two opposite forces give rise to an U-shaped relationship between the monopoly power and the risk of bank failure.

My analysis intermediaries operate under an implicit no-default constraint and hence there are no bank failures. This does not mean, however, that the issue of **financial stability** does not emerge because the amount of intermediation (and aggregate investment) depends positively on banks' equity (i.e., accumulated earnings in my model).⁷ If aggregate level of equity is low then so is the resource reallocation and aggregate investment. Financial shocks drain banks' equity, lead to lower aggregate investment and recessions. If financial intermediaries' have monopoly power then they are able to accumulate an equity cushion that buffers potential financial shocks. So the first part of the "trade-off" in my model is similar to the force described by [Martinez-Miera and Repullo (2010)]. The second part, however, has nothing to do with investment risk choice made by firms. It is a standard result that makes monopolistic banks less favorable: monopolistic intermediaries channel less resources and they impose higher spreads than competitive bankers. As a result, in normal times level of aggregate investment is lower which in turn decreases capital stock and output.

To my best knowledge, there are no papers that describe the impact of financial intermediaries' market structure on the real economy in the context of business cycle fluctuations. This work is intended to fill in this gap by incorporating a simple banking system into otherwise standard neoclassical

⁷This means that the notion of financial stability is associated with the volatility of resources channeled by intermediaries in my analysis and not with the bank failures.

framework. Additionally, the analysis captures both dynamic and general equilibrium effects that were ignored in some articles cited above that have a static or a partial-equilibrium character.

Dynamic equilibrium models. There is an immense literature on financial frictions and the role of the banking sector in the RBC framework. I would like to concentrate on two articles that are closely related to my work (i.e., they use similar formalization techniques to address the issues of trades in capital and the role of banks in the economy).

Firstly, my model builds on the construction presented by [Kiyotaki and Moore (2012)]. To give rise to trade in assets (capital), [Kiyotaki and Moore (2012)] split the population of entrepreneurs into two segments: investors (that hold investment opportunities) and those who do not have such opportunities in the current period. Investors issue equity claims (that entitle their holders to capital income streams) to finance their projects and non-entrepreneurs purchase those claims as they cannot invest. This division of population gives rise to trade in assets. I use a similar construction to generate the endogenous reallocation of resources. There is, however, a fundamental difference between their work and mine. In [Kiyotaki and Moore (2012)] agents do not need services provided by intermediaries to sell/purchase capital whereas in my model only banks can channel capital between entrepreneurs and hence they are central actors in the drama.

From the technical point of view, the most closely related article to mine is [Bigio (2015b)]. Similarities between my work and [Bigio (2015b)] entail: the presence of two types of entrepreneurs (consumption goods and investment goods producers) and banks that transfer capital sold by investment goods producers to consumption goods producers.

There are, however significant differences: I do not include asymmetric information about capital quality that gives rise to multiplicity of equilibria and the "rocking boat" dynamics that follows financial crises in [Bigio (2015b)]. To avoid the problem of multiplicity and to generate strictly increasing supply of capital in the model, I assume that investment goods producers have different productivity levels and hence some of them are more willing to sell their capital than the others. Another difference is associated with the source of aggregate uncertainty. In [Bigio (2015b)], there are two aggregate shocks: the standard productivity shock and the one that affects capital depreciation. I do not have the shock that affects the technology level in my model and the only aggregate shock influences the demand for capital sold by intermediaries: sudden drops in demand induce balance sheet losses of banks and drain their equity. Not only have these disturbances a clear interpretation (shifts in preferences/panics in capital markets) but also admit a tractable and illustrative analysis.

1.3 Economy with perfectly competitive intermediaries

In this section I study the allocation generated by economy with perfectly competitive banking sector.

1.3.1 Environment

Time. Time is infinite and divided into discrete periods. Each period consists of two subsequent stages.

Agents. The model is populated by three classes of agents: infinitely-lived entrepreneurs (that are called producers as well), infinitely-lived financial intermediaries (called banks, too) and workers. First two populations have measures normalized to one. Population of workers has measure L . Financial intermediaries are identical and there are two types of entrepreneurs: consumption goods producers and investment goods producers that have measures π_C and $\pi_I = 1 - \pi_C$, respectively.⁸

Shocks. There is one aggregate shock: an i.i.d. shock $Z_t \in \mathbb{R}_+$. It affects demand (of c-producers) for capital transferred by intermediaries and gives rise to portfolio risk faced by banks. Moreover, there is an idiosyncratic uncertainty faced by entrepreneurs: at the beginning of first stage, entrepreneurs are randomly segmented into two subgroups: c-producers and i-producers. This division generates two separate populations of entrepreneurs: those who consider selling their capital to finance their investment projects (i-producers) and those who want to purchase capital (c-entrepreneurs). Additionally, every i-entrepreneur draws the productivity level that is associated with his investment opportunity which is an additional source of idiosyncratic uncertainty faced by producers. It will be clear later that introducing investment opportunities of different productivity levels gives rise to a differentiable and monotonically increasing supply of capital. In contrast to i-entrepreneurs, all c-producers operate identical production technology.

In what follows I assume that idiosyncratic shocks are independent of individual capital holdings which greatly simplifies the analysis - it enables me to aggregate individual demands and supplies of capital. It is because aggregates become independent of distribution of capital.⁹

Goods, technologies and trade. There are two types of goods: capital goods and consumption goods and two production factors: capital and labor. C-entrepreneurs use their capital holdings k and hire l workers (that are paid wage w) to produce consumption goods. They operate the Cobb-Douglas technology $A_C k^\alpha l^{1-\alpha}$ where A_C is technology level that is equal across c-entrepreneurs. C-producers are not able to manufacture capital goods. Since their capital holdings depreciate (this occurs between periods at rate δ), they are willing to increase it and hence they have incentives to purchase capital.

Consumption goods can be transformed into capital by i-entrepreneurs. They have an access to a linear technology that generates $A_I i$ capital goods (that increase the i-producers capital holdings in the next period) out of i consumption goods. I assume that A_I varies across i-producers. In particular A_I is drawn from the probability distribution described by a continuous density function $f(A_I)$ that satisfies $\mathbb{E}A_I < +\infty$ and $\text{supp}(f) = \mathbb{R}_+$.¹⁰ By \mathbb{P}_{A_I} I denote the probability measure associated with A_I . Amount of consumption goods used to generate capital is called investment.¹¹ I-entrepreneurs are unable, however, to use their capital holdings to produce consumption goods. To get them, they have to sell their capital holdings.

Workers are identical. Each worker supplies one unit of labor inelastically and I assume that they do not have access to financial markets so they simply consume their wages each period. I introduce workers to the model to guarantee that c-producers' profits are linear in capital holdings (that enables

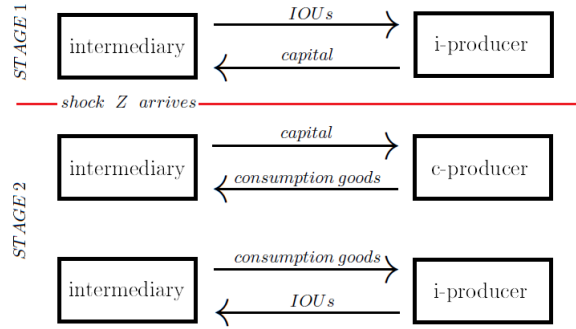
⁸I refer to them as c-producers/c-entrepreneurs and i-producers/i-entrepreneurs, too.

⁹There are some additional assumptions that are necessary to obtain this aggregation result that are discussed later.

¹⁰I assume that f is continuous because it guarantees that aggregate supply of capital is a smooth (differentiable) function.

¹¹For example, at individual level, i is called investment and $A_I i$ is final output of capital goods.

Figure 2: Financial intermediation



me to derive analytic formulas for c-entrepreneurs' policies) and to ensure that my environment is stationary (since the production technology of consumption goods is concave in capital given the Cobb-Douglas technology).

C-entrepreneurs and i-entrepreneurs cannot trade capital and consumption goods directly, they have to use services provided by banks instead. In equilibrium, during the first stage, intermediaries buy capital from i-producers (capital sellers) at price q_S and they finance their purchases with riskless IOUs that they issue. At the same time, c-producers generate consumption goods. At the end of the first stage value of aggregate shock Z is realized. During the second stage, banker transfers capital to capital buyers. Intermediary gets q_B consumption goods produced by capital purchasers (c-entrepreneurs) for one unit of capital resold by banks. At the end of the second stage banker transfers consumption goods to sellers to settle their debt (IOUs). All agents consume at the end of the second stage and i-entrepreneurs produce capital using consumption goods received from banks as an input. The sequence of transactions is presented in Figure 2. I assume that intermediaries cannot default on their debt (i.e., IOUs held by capital sellers) and that they are not able to store capital.¹² The latter implies that they transfer the total amount of capital purchased from i-entrepreneurs.¹³ On the other hand, banks have technology to store consumption goods so they are able to accumulate equity over time (which means that physically it is a stock of consumption goods). The only storage technology available to producers is capital storage technology. Notice that the no-default constraint has an implication for the character of the contract between capital sellers (i-entrepreneurs) and intermediaries: it resembles a standard deposit because it does not depend on changes in aggregate conditions.

Preferences. Workers, bankers and i-entrepreneurs have preferences over lifetime consumption

¹²If one relaxes this assumption then the portfolio risk faced by banks decays: if market conditions are poor then banks decide to store capital and they sell it later. This implies that they do not generate large losses after adverse aggregate shocks, their equity is not drained and slow recoveries after financial disturbances are eliminated. This in turn means that the dynamics that is typical for financial/banking crises cannot take place in such a model. Therefore, the assumption that intermediaries cannot store capital is essential for generating financial crisis episodes in my environment.

¹³This assumption makes their portfolio marked-to-market.

streams $\{c_t\}_{t=0}^{+\infty}$ described by:

$$\mathbb{E}_0 \left(\sum_{t=0}^{+\infty} \beta^t u(c_t) \right),$$

where u is a strictly increasing and strictly concave function of c_t and $0 < \beta < 1$ is their discount factor. Observe, that it is common for models that can be found in the literature about financial markets to assume linear preferences for intermediaries. However, concave intermediary's utility function u can be justified by dividend-smoothing motives (applied to entrepreneurs by [Jermann and Quadrini (2012)]). Recent use of concave preferences of bankers can be found in [Brunnermeier and Sannikov (2014)]. I make this assumption because it guarantees the existence of interior solution to the banker's problem and hence it enables comparative statics exercises.

C-producers have preferences that depend on the aggregate shock Z_t :

$$\mathbb{E}_0 \left(\sum_{t=0}^{+\infty} \beta^t \cdot Z_t \cdot u(c_t) \right).$$

This dependence is introduced to give rise to shifts in demand for capital purchased from intermediaries. If Z_t is high then c-entrepreneurs value consumption more and their demand for capital drops.

Assumptions made in this section are discussed in a more detailed way in Appendix A.

1.3.2 Optimization problems

Workers. As it has been mentioned before, workers are hand-to-mouth. This means that they simply consume their wages w_t :

$$c_t = w_t. \tag{1}$$

It is assumed that their utility function has a logarithmic specification.

I-producers. I start with the dynamic problem of i-producer that begins period with capital holdings k and is affected by productivity shock A_I . From the description of the intermediation process we know that it makes its decisions in the first stage. The corresponding Bellman equation reads:

$$\begin{aligned} V^I(k, K, E, A_I) = \max_{c > 0, i \geq 0, k_S > 0, k' > 0} \{ & \log(c) + \beta \mathbb{E}_{Z, Z', A'_I} (\pi_I \cdot V^I(k', K', E', A'_I) \\ & + \pi_C \cdot V^C(k', K', E', Z') | K, E) \}. \end{aligned} \tag{2}$$

subject to :

$$\begin{cases} c + i = q_S(K, E) \cdot k_S, \\ k' = A_I \cdot i + (1 - \delta)(k - k_S), \\ E' = E'(K, E, Z), \\ K' = K'(K, E), \end{cases}$$

where V^I is value function associated with the dynamic maximization problem of i-entrepreneur and V^C is value function associated with the problem of c-producer and prime symbols denote next period values of variables. Observe that arguments of V^I and V^C are different: it is because i-entrepreneur makes its decisions (about selling capital) in the first stage, before realization of Z and because c-producers do not face idiosyncratic uncertainty associated with their productivity levels. By E I denote the aggregate stock of banks' equity (that alternatively can be treated as reserves of liquid assets held by banks, too). First equation that determines the set of possible actions is the budget constraint of i-entrepreneur: it says that i-producer sells k_S units of its capital holdings at price $q_S(K, E)$ and uses the proceedings (consumption goods) for investment i and consumption c . Second constraint is the law of motion for individual capital holdings. Observe that amount of capital generated out of i consumption goods depends on the productivity level A_I . Expression $(1 - \delta)(k - k_S)$ denotes the unsold capital that depreciates at rate δ . Third and fourth constraints describe perceived laws of motion for aggregate banks' equity E and aggregate capital K (i.e., it captures an implicit assumption about agents' rational expectations).

Notice that I assume the logarithmic form of utility. I will show that this assumption guarantees that entrepreneurs' (intermediaries') policy functions are linear in capital holdings k (or bank's equity holdings e). This coupled with assumption about capital holdings' independence of productivity shocks means that distribution of entrepreneurs' capital holdings is not a state variable.

Observe that if i-entrepreneur's productivity A_I is sufficiently high then he may decide to sell all his capital holdings k to finance his investment i (and consumption). On the other hand, if it is low enough, then i-producer decides to reduce the amount of capital that is sold - k_S and sets $i = 0$. The following lemma formalizes this intuition:

Lemma 1. *Suppose that i and k_S solve 2. If $A_I \geq A_I^*(q_S)$ then $i > 0$ and $k_S = k$. If $A_I < A_I^*(q_S)$ then $i = 0$ and $0 < k_S < k$.*

Lemma 1 is proved in Appendix B¹⁴. The critical value $A_I^*(q_S)$ satisfies:

$$A_I^*(q_S) = \frac{1 - \delta}{q_S(K, E)}.$$

Lemma 1 is useful as it allows me to split the i-producer's problem 2 into two separate problems that admit interior solutions.

The first problem pertains to i-entrepreneur that has productivity A_I that satisfies $A_I \geq A_I^*(q_S)$:

$$V^{IP}(k, K, E, A_I) = \max_{c > 0, i \geq 0, k' > 0} \{ \log(c) + \beta \mathbb{E}_{Z, Z', A'_I} (\pi_I \cdot \mathbb{P}_{A_I} (A_I \geq A_I^*(q'_S)) \cdot V^{IP}(k', K', E', A'_I) \}$$

¹⁴ All proofs are moved into Appendix B.

$$+\pi_I \cdot \mathbb{P}_{A_I} (A_I < A_I^*(q'_S)) \cdot V^{I_0}(k', K', E') + \pi_C \cdot V^C(k', K', E', Z')|K, E)\}.$$

subject to :

$$\begin{cases} c + i = q_S(K, E) \cdot k, \\ k' = A_I \cdot i, \\ E' = E'(K, E, Z), \\ K' = K'(K, E), \end{cases}$$

where V^{I_P} is value function associated with the problem of i-entrepreneur whose current productivity level is $A_I \geq A_I^*(q_S)$ and who produces new capital, V^{I_0} is value function that corresponds to the problem of i-producer that has a relatively low productivity (i.e., $A_I < A_I^*(q_S)$) and it sets its investment at the level $i = 0$. Budget constraint indicates that i-entrepreneur with $A_I \geq A_I^*(q_S)$ sells his entire capital k and law of motion for his capital shows that his future capital holdings come entirely from creation of new capital.

The second problem corresponds to i-producer that has low productivity: $A_I < A_I^*(q_S)$. Budget constraint shows that $i = 0$ and entrepreneur does not sell his entire capital holdings as $k' > 0$. According to the law of motion, unsold capital depreciates and becomes producer's capital holdings in the next period:

$$V^{I_0}(k, K, E) = \max_{c>0, k_S>0, k'>0} \left\{ \log(c) + \beta \mathbb{E}_{Z, Z', A'_I} (\pi_I \cdot \mathbb{P}_{A_I} (A_I \geq A_I^*(q'_S)) V^{I_P}(k', K', E', A'_I) \right. \\ \left. + \pi_I \cdot \mathbb{P}_{A_I} (A_I < A_I^*(q'_S)) V^{I_0}(k', K', E') + \pi_C \cdot V^C(k', K', E', Z')|K, E) \right\}.$$

subject to :

$$\begin{cases} c = q_S(K, E) \cdot k_S, \\ k' = (1 - \delta) [k - k_S], \\ E' = E'(K, E, Z), \\ K' = K'(K, E). \end{cases}$$

C-producers. This group of entrepreneurs makes decisions in the second stage, after the realization of aggregate shock Z . They choose their consumption, capital purchases and number of workers hired:

$$V^C(k, K, E, Z) = \max_{c>0, k_B \in \mathbb{R}, k'>0, l>0} \left\{ Z \cdot \log(c) + \beta \mathbb{E}_{Z', A'_I} (\pi_I \cdot \mathbb{P}_{A_I} (A'_I \geq A_I^*(q'_S)) V^{I_P}(k', K', E', A'_I) \right. \\ \left. + \pi_I \cdot \mathbb{P}_{A_I} (A'_I < A_I^*(q'_S)) V^{I_0}(k', K', E') + \pi_C \cdot V^C(k', K', E', Z')|K, E) \right\}. \quad (3)$$

subject to :

$$\begin{cases} c + q_B(K, E, Z)k_B = A_C k^\alpha l^{1-\alpha} - w(K) \cdot l, \\ k' = (1 - \delta) [k + k_B], \\ E' = E'(K, E, Z), \\ K' = K'(K, E), \end{cases}$$

where $q_B(K, E, Z)$ is price at which c-entrepreneurs buy assets from intermediaries and k_B is amount of purchased capital. We will see that in equilibrium $k_B > 0$. Observe that Z affects the c-producer's preferences which gives rise to changes in demand for asset purchases k_B .

Since l enters only the RHS of c-producer's budget constraint, problem 3 can be analyzed in two stages: first, I maximize c-producer's profits $A_C k^\alpha l^{1-\alpha} - w(K) \cdot l$ with respect to l and then I solve the maximization problem with respect to the remaining variables: $c > 0$, $k_B \in \mathbb{R}$, $k' > 0$. The value of l that solves the first maximization problem satisfies:

$$l^* = \left(\frac{(1 - \alpha)A_C}{w(K)} \right)^{\frac{1}{\alpha}} \cdot k. \quad (4)$$

Plugging this solution l^* into dynamic problem 3 yields:

$$\begin{aligned} V^C(k, K, E, Z) = & \max_{c > 0, k_B \in \mathbb{R}, k' > 0} \left\{ Z \cdot \log(c) + \beta \mathbb{E}_{Z', A'_I} \left(\pi_I \cdot \mathbb{P}_{A_I} (A'_I \geq A_I^*(q'_S)) V^{I_P}(k', K', E', A'_I) \right. \right. \\ & \left. \left. + \pi_I \cdot \mathbb{P}_{A_I} (A'_I < A_I^*(q'_S)) V^{I_0}(k', K', E') + \pi_C \cdot V^C(k', K', E', Z') | K, E \right) \right\}. \end{aligned}$$

subject to :

$$\begin{cases} c + q_B(K, E, Z)k_B = G(K) \cdot k, \\ k' = (1 - \delta) [k + k_B], \\ E' = E'(K, E, Z), \\ K' = K'(K, E), \end{cases}$$

where $G(K)$ satisfies $G_K < 0$.¹⁵ This means that the RHS of c-entrepreneur's budget constraint is linear in his capital holdings k . This property is useful in the next subsection in which I characterize policy rules and value functions of entrepreneurs.

Characterization of decision rules. I will show that given logarithmic preferences and budget constraints that are linear in asset holdings, policy functions associated with maximization problems listed above are linear in producer's capital holdings k . This enables me to aggregate the decisions made

¹⁵The exact formula for $G(A_C, w(K))$ is:

$$\begin{aligned} G(K) &= (1 - \alpha)^{\frac{1}{\alpha}} \cdot \left(\frac{\alpha}{1 - \alpha} \right) \cdot A_C^{\frac{1}{\alpha}} \cdot w(K)^{1 - \frac{1}{\alpha}} \\ &= \alpha A_C (\pi_C \cdot [K/L])^{\alpha - 1} \end{aligned}$$

where I have used equation 12.

by all producers within each segment (of i-entrepreneurs and c-entrepreneurs) and derive aggregate supply of capital and aggregate demand for assets. The following proposition characterizes policy functions:

Proposition 1. *Decision rules and value function of an i-producer that has productivity level $A_I < A_I^*(q_S)$ are: $c = \frac{\phi}{1+\phi}\omega_{I_0}$, $k' = \frac{1}{1+\phi}\frac{(1-\delta)\omega_{I_0}}{q_S}$, $V^{I_0} = \Psi^{I_0}(K, E) + \left(1 + \frac{1}{\phi}\right)\log \omega_{I_0}$ where $\omega_{I_0} = q_S k$ and $\phi = \frac{1-\beta}{\beta(\pi_I + \pi_C \cdot \mathbb{E}Z)}$. Decision rules and value function of an i-producer that has productivity level $A_I \geq A_I^*(q_S)$ are: $c = \frac{\phi}{1+\phi}\omega_{I_P}$, $k' = \frac{1}{1+\phi}A_I\omega_{I_P}$, $V^{I_P} = \Psi^{I_P}(K, E, A_I) + \left(1 + \frac{1}{\phi}\right)\log \omega_{I_P}$ where $\omega_{I_P} = q_S k$. Decision rules and value function of a c-producer are: $c = \frac{\phi Z}{1+\phi Z}\omega_C$, $k' = \frac{1}{1+\phi Z}\frac{(1-\delta)\omega_C}{q_B}$, $V^C = \Psi^C(K, E, Z) + \left(Z + \frac{1}{\phi}\right)\log \omega_C$ where $\omega_C = (G(K) + q_B)k$.*

Proposition 1 enables the derivation of aggregate demand for labor, aggregate supply and aggregate demand for capital.

Aggregate demand for labor. Aggregation of 4 across the c-producers yields:

$$L_D(w(K), K) = \left(\frac{(1-\alpha)A_C}{w(K)} \right)^{\frac{1}{\alpha}} \cdot \pi_C \cdot K.$$

Function L_D is decreasing in wages and increases with technology level A_C .

Aggregate supply of capital. First observe that i-entrepreneurs with higher productivity (i.e., those $A_I \geq A_I^*(q_S)$) sell their entire capital. The total measure of those agents is $\pi_I \cdot \mathbb{P}_{A_I}(A_I \geq A_I^*(q_S))$. There is $\pi_I \cdot \mathbb{P}_{A_I}(A_I < A_I^*(q_S))$ of i-producers that set $i = 0$. Individual supply of capital of the latter is:

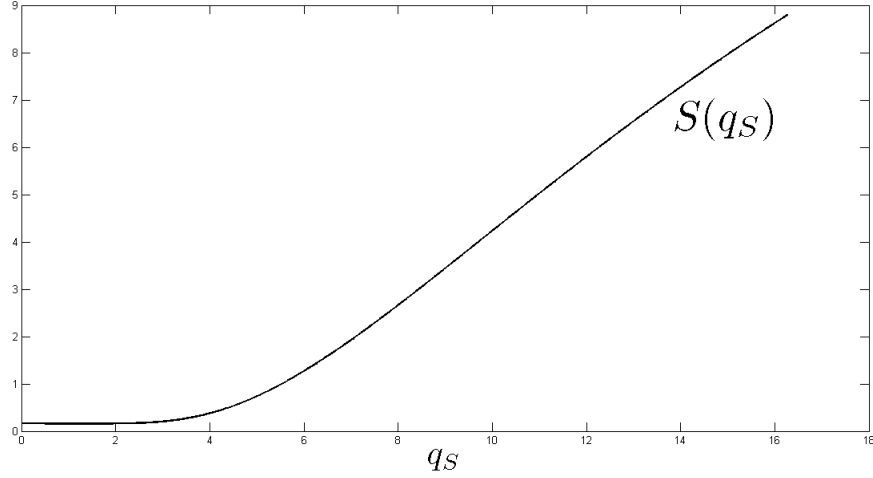
$$\begin{aligned} k_S &= k - \frac{k'}{1-\delta} \\ &= k - \frac{1}{1-\delta} \cdot \frac{1}{1+\phi} \cdot \frac{(1-\delta)}{q_S} \cdot q_S k \\ &= \frac{\phi}{1+\phi} k \end{aligned}$$

where I have used Proposition 1. Since the idiosyncratic shock that divides the pool of entrepreneurs into i-producers and c-producers is independent of individual capital holdings (shock A_I satisfies this property, too) then I get the following formula for aggregate supply of capital $S(q_S)$:

$$S(q_S, K) = \left\{ \mathbb{P}_{A_I}(A_I < A_I^*(q_S)) \frac{\phi}{1+\phi} + \mathbb{P}_{A_I}(A_I \geq A_I^*(q_S)) \right\} \cdot \pi_I \cdot K. \quad (5)$$

Since $0 < \frac{\phi}{1+\phi} < 1$ then $S_{q_S}(q_S, K) > 0$. This is because as q_S decreases then $A_I^*(q_S) = \frac{1-\delta}{q_S}$ falls and there is more i-entrepreneurs that sell their entire capital holdings. Observe that because density $f(A_I)$ was assumed to be continuous then by the Fundamental Theorem of Calculus $S_{q_S}(q_S, K)$ exists and is continuous in q_S . Notice that $\lim_{q_S \rightarrow 0} S(q_S, K) = \frac{\phi}{1+\phi} \cdot \pi_I \cdot K$, $\lim_{q_S \rightarrow +\infty} S(q_S, K) = \pi_I \cdot K$ for any value of K . The case of $\lim_{q_S \rightarrow 0} S(q_S, K) > 0$ seems to be surprising. This happens because if $q_S \rightarrow 0$ then $A_I^*(q_S) \rightarrow +\infty$ and measure of i-producers that set $i = 0$ converges to π_I . Since they are not able to produce consumption goods they sell a non-zero proportion $\frac{\phi}{1+\phi}$ of their capital holdings.

Figure 3: Aggregate supply of capital (fixed K)



It is because they need to consume due to the logarithmic utility function. Figure 3 illustrates the aggregate supply of capital.¹⁶

Aggregate demand for capital. Individual demand for capital of c-entrepreneur can be derived using the formula for k' in Proposition 1:

$$\begin{aligned} k_B &= \frac{k'}{1 - \delta} - k \\ &= k \cdot \left[\frac{1}{1 + \phi Z} \cdot \frac{G(K)}{q_B} + \frac{1}{1 + \phi Z} - 1 \right]. \end{aligned}$$

Again, I use the assumption about independence of idiosyncratic shocks of individual capital holdings which means that the aggregate demand for capital has the following form:

$$D(q_B, K, Z) = \left[\frac{1}{1 + \phi Z} \cdot \frac{G(K)}{q_B} + \frac{1}{1 + \phi Z} - 1 \right] \cdot \pi_C \cdot K. \quad (6)$$

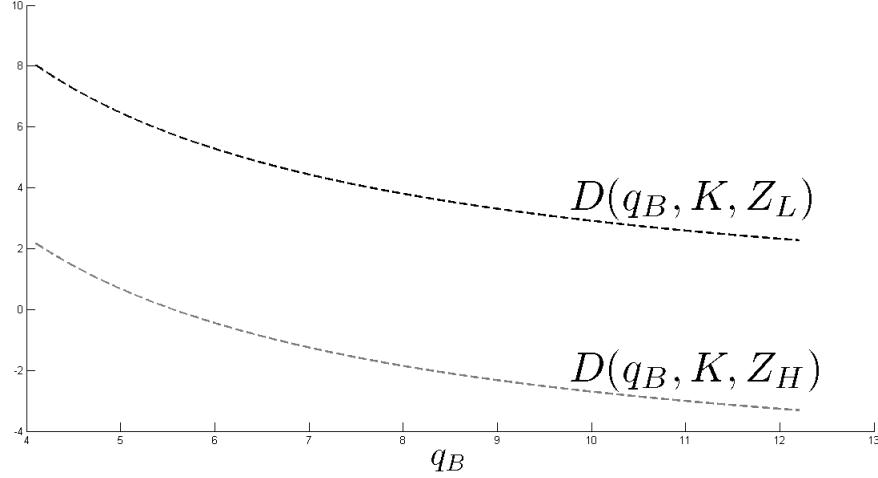
It is clear that: $D_{q_B} < 0$ and $D_Z < 0$. The latter implies that increase in Z translates into lower amount of assets demanded by c-producers. This in turn decreases the price of assets that are held by intermediaries (since amount of capital is fixed as intermediaries buy assets from i-entrepreneurs during the first stage as it is shown in Figure 2).

Observe that $\lim_{q_B \rightarrow 0} D(q_B, K, Z) = +\infty$ and $\lim_{q_B \rightarrow +\infty} D(q_B, K, Z) = -\frac{\phi Z}{1 + \phi Z} \cdot \pi_C \cdot K < 0$. Additionally, if $q_B = q_B^0(K, Z) = \frac{G(K)}{\phi Z}$ then $D(q_B, K, Z) = 0$. Aggregate demand function is presented in Figure 4.

Aggregate law of motion for capital. To derive the aggregate law of motion for capital let

¹⁶To plot Figures 3, 4, 5, 6 I use the following parameter values $\pi_I = 0.2$, $A_C = 1$, $a = b = 0.3$, $Z_L = 1$, $Z_H = 2$, $\beta = 0.99$, $L = 100$, $\delta = 0.025$, $\alpha = 0.33$, $L = 100$, $\pi(Z_H) = 0.2$, $\epsilon = 1.1$. For Figures 5, 6 and of the fixed levels of K or E is the long run level of K (or E) after a long time of "normal times" ($Z = Z_L$).

Figure 4: Aggregate demand for capital (for low and high realizations of Z , fixed K)



us start with investment decision made by i-entrepreneurs whose productivity satisfies $A_I \geq A_I^*(q_S)$. Amount of new capital generated by such an entrepreneur is (according to Proposition 1):

$$A_I \cdot i = k' = \frac{1}{1+\phi} \cdot A_I \cdot q_S \cdot k.$$

Aggregation across different values of A_I and individual capital holdings yields:¹⁷

$$I(q_S, K) = \left[\int_{A_I^*(q_S)}^{+\infty} \frac{1}{1+\phi} \cdot A_I \cdot q_S \cdot f(A_I) dA_I \right] \pi_I \cdot K. \quad (7)$$

Let us analyze the effects of q_S on aggregate effective investment $I(q_S, K)$.¹⁸ Formula 7 shows that there are two forces at play. First, an increase in q_S causes a drop in $A_I^*(q_S)$. This means that more i-producers decide to sell their entire capital and invest a part $\frac{1}{1+\phi}$ of their proceedings (this is the extensive margin). Second, it boosts i-entrepreneurs wealth $\omega_{I_P} = q_S k$, so that they are able to get more consumption goods in exchange for the capital that is sold. Since their technology uses consumption goods to generate capital and since the proportion of goods that are invested $\frac{1}{1+\phi}$ is

¹⁷An alternative, equivalent formulation of $I(q_S, K)$ is:

$$\begin{aligned} I(q_S, K) &= \pi_I \cdot \mathbb{P}_{A_I} (A_I \geq A_I^*(q'_S)) \cdot \mathbb{E} \left(\frac{1}{1+\phi} \cdot A_I \cdot q_S \mid A_I \geq A_I^*(q'_S) \right) \cdot K \\ &= \pi_I \cdot \mathbb{P}_{A_I} (A_I \geq A_I^*(q'_S)) \cdot \frac{1}{\mathbb{P}_{A_I} (A_I \geq A_I^*(q'_S))} \int_{A_I^*(q_S)}^{+\infty} \frac{1}{1+\phi} \cdot A_I \cdot q_S \cdot f(A_I) dA_I \cdot K \\ &= \left[\int_{A_I^*(q_S)}^{+\infty} \frac{1}{1+\phi} \cdot A_I \cdot q_S \cdot f(A_I) dA_I \right] \pi_I \cdot K. \end{aligned}$$

¹⁸Observe that $I(q_S, K)$ is the aggregate amount of capital produced by i-entrepreneurs and not the amount of consumption goods used for producing capital.

constant, then it leads to increase in $I(q_S, K)$ (intensive margin). This means that q_S depends crucially on the condition of banks' balance sheets and hence the discussed effects constitute a mechanism through which financial disturbances are transmitted to real economy.

Since the remaining capital depreciates at rate $0 < \delta < 1$ then the aggregate law of motion for K is:

$$K' = (1 - \delta)K + I(q_S, K). \quad (8)$$

1.3.3 Intermediaries

Banks are competitive and take prices q_S and q_B as given. They begin the period with e consumption goods (called equity) - this amount is determined by their decision made in the previous period.¹⁹ In the first stage they decide how much capital k_F they are going to transfer and during the second stage (after the realization of Z) they choose how much consumption goods c they consume and how much becomes their equity in the next period - e' . Since decisions are made in both stages, I need to specify two maximization problems. In the first stage bank solves:

$$W_1(e, K, E) = \max_{k_F} \mathbb{E}_Z (W_2(k_F, e, K, E, Z)), \quad (9)$$

where W_1 is value function corresponding to the maximization problem solved in the first stage and W_2 is associated with the second stage problem that reads:

$$W_2(k_F, e, K, E, Z) = \max_{c, e'} \{ \log(c) + \beta W_1(e', K', E') \} \quad (10)$$

subject to :

$$\begin{cases} c + e' = e + (q_B(K, E, Z) - q_S(K, E)) \cdot k_F, \\ E' = E'(K, E, Z), \\ K' = K'(K, E). \end{cases}$$

The first constraint is bank's budget constraint and it captures the implicit assumption that banks cannot default on their liabilities $q_S(K, E)k_F$.

It will become clear later (see Lemma 2) that in equilibrium both positive and negative spreads $q_B(K, E, Z) - q_S(K, E)$ are possible (where the sign depends on the realization of Z). This in turn implies that the decision about k_F is risky: not only does $q_B(K, E, Z)$ vary with Z but also $q_S(K, E) \cdot k_F$ that has to be repaid to capital sellers at the end of the second stage is unaffected by the realization of aggregate uncertainty. This gives rise to the portfolio risk that is faced by intermediaries.

To get analytic formulas for policies and value functions the following condition needs to be satisfied:

A1 Aggregate shock Z takes two values: $Z \in \{Z_L, Z_H\}$. Probabilities $\mathbb{P}(Z = Z_L) = \pi(Z_L)$

¹⁹ An alternative interpretation for this variable is amount of liquid reserves held by intermediaries.

and $\mathbb{P}(Z = Z_H) = \pi(Z_H)$ satisfy $\pi(Z_L) + \pi(Z_H) = 1$.

This assumption is made for clarity of exposition (it is easier to analyze spreads in this case) and it simplifies the problem of uniqueness of the RCE with competitive banking sector. Observe, that since $D_Z < 0$ then the state in which $Z = Z_H$ can be referred to as "crisis" in which demand for capital held by intermediaries drops (which causes a decrease in price $q_B(K, E, Z)$) and hence the value of banks' assets $q_B(K, E, Z) \cdot k_F$ falls.

Additionally, assumption A1 allows to transform 10 into a standard consumption-savings problem (and hence admits analytical solution). It is because A1 implies that the optimal choice of k_F is a linear function of e , i.e.: $k_F = \Phi(K, E) \cdot e$. This means that the budget constraint in problem 10 takes the form that is analogous to the one in a standard consumption-savings problem:

$$c + e' = (1 + (q_B - q_S) \cdot \Phi(K, E)) \cdot e.$$

This is the main intuition behind the proof of the following proposition:

Proposition 2. *If A1 holds then decision rules and value function of intermediary are: $c = (1 - \beta)\omega_F$, $e' = \beta\omega_F$, $W_2 = \Psi^F(K, E, Z) + \frac{1}{1-\beta} \log \omega_F$, $k_F = \Phi(K, E) \cdot e$, where $\omega_F = e + (q_B - q_S)k_F$.*

Proposition 2 will be useful when I characterize and investigate the equilibrium allocation with competitive banking sector. The exact analytic formula for $\Phi(K, E)$ is presented in Appendix B. It is easy to check that $k_F = \Phi(K, E) \cdot e$ can be rewritten as:

$$\begin{aligned} & \pi(Z_L) \cdot \frac{q_B(K, E, Z_L) - q_S(K, E)}{e + [q_B(K, E, Z_L) - q_S(K, E)] k_F} \\ & + \pi(Z_H) \cdot \frac{q_B(K, E, Z_H) - q_S(K, E)}{e + [q_B(K, E, Z_H) - q_S(K, E)] k_F} = 0, \end{aligned} \quad (11)$$

which is the FOC that describes the optimal choice of k_F during the first stage.

1.3.4 Equilibrium

In this subsection I present the definition of recursive competitive equilibrium. To distinguish between consumption and capital choices of different types of agents, I use subscripts I_P , I_0 , C and F (for i-producer with $A_I \geq A_I^*(q_S)$, i-producer with $A_I < A_I^*(q_S)$, c-producer and financial intermediary, respectively).

Definition 1. Recursive Competitive Equilibrium with competitive banking sector consists of: pricing functions $q_B(K, E, Z)$, $q_S(K, E)$, $w(K)$ perceived law of motion for intermediaries' equity $E'(K, E, Z)$ and aggregate capital $K'(K, E)$, decision rules $k_F(e, K, E)$, $e'(e, K, E, Z)$, $c_F(e, K, E, Z)$, $c_C(k, K, E, Z)$, $k'_C(k, K, E, Z)$, $k_B(k, K, E, Z)$, $l(k, K, E, Z)$, $c_{I_P}(k, K, E, A_I)$, $k'_{I_P}(k, K, E, A_I)$, $i(k, K, E, A_I)$, $c_{I_0}(k, K, E)$, $k'_{I_0}(k, K, E)$, $k_S(k, K, E)$, associated value functions $W_1(e, K, E)$, $W_2(k_F, e, K, E, Z)$, $V^C(k, K, E, Z)$, $V^{I_P}(k, K, E, A_I)$, $V^{I_0}(k, K, E)$ and stochastic processes that determine the evolution of Z , A_I and producer's type over time such that:

1) Decision rules $k_F(e, K, E)$, $e'(e, K, E, Z)$, $c_F(e, K, E, Z)$ and $W_1(e, K, E)$, $W_2(k_F, e, K, E, Z)$ solve the the dynamic financial intermediary given: $q_B(K, E, Z)$, $q_S(K, E)$, $E'(K, E, Z)$, $K'(K, E)$ and stochastic processes,

2) Decision rules $c_C(k, K, E, Z)$, $k'_C(k, K, E, Z)$, $k_B(k, K, E, Z)$, $l(k, K, E, Z)$ and value functions $V^C(k, K, E, Z)$, $V^{IP}(k, K, E, A_I)$, $V^{I_0}(k, K, E)$ solve the the dynamic problem of c-producer given: $q_B(K, E, Z)$, $w(K)$, $E'(K, E, Z)$, $K'(K, E)$ and stochastic processes,

3) Decision rules $c_{IP}(k, K, E, A_I)$, $k'_{IP}(k, K, E, A_I)$, $i(k, K, E, A_I)$ and value functions $V^C(k, K, E, Z)$, $V^{IP}(k, K, E, A_I)$, $V^{I_0}(k, K, E)$ solve the the dynamic problem of i-producer for which $A_I \geq A_I^*(q_S(K, E))$ given $q_S(K, E)$, $E'(K, E, Z)$, $K'(K, E)$ and stochastic processes,

4) Decision rules $c_{I_0}(k, K, E)$, $k'_{I_0}(k, K, E)$, $k_S(k, K, E)$ and value functions $V^C(k, K, E, Z)$, $V^{IP}(k, K, E, A_I)$, $V^{I_0}(k, K, E)$ solve the the dynamic problem of i-producer for which $A_I < A_I^*(q_S(K, E))$ given $q_S(K, E)$, $E'(K, E, Z)$, $K'(K, E)$ and stochastic processes,

5) Consistency conditions hold: $e'(e, K, E, Z) = E'(K, E, Z)$ and $K'(K, E) = (1-\delta)K + I(q_S(K, E), K)$, where:

$$I(q_S(K, E), K) = \left[\int_{A_I^*(q_S(K, E))}^{+\infty} i(k, K, E, A_I) \cdot f(A_I) dA_I \right] \pi_I.$$

6) Markets clear, i.e.: $k_F(e, K, E) = S(q_S(K, E), K)$, $k_F(e, K, E) = D(q_B(K, E, Z), K, Z)$, $L_D(w(K), K) = L$.

Characterization. Suppose that values are K and E are given. I will discuss how can one obtain K' and E' using model's equations and the current realization of Z .

First, notice that we can use the labor market clearing condition $L_D(w(K), K) = L$ to get the expression for wages:

$$w(K) = (1 - \alpha) \cdot A_C \cdot (\pi_C \cdot K)^\alpha \cdot L^{-\alpha}. \quad (12)$$

Let us combine bank's FOC 11 with reformulated market clearing conditions and the consistency condition $E = e$ (I suppress arguments of policy and pricing functions to economize on notation):²⁰

$$\begin{aligned} \pi(Z_L) \cdot \frac{D^{-1}(S(q_S, K), K, Z_L) - q_S}{E + [D^{-1}(S(q_S, K), K, Z_L) - q_S] S(q_S)} \\ \pi(Z_H) \cdot \frac{D^{-1}(S(q_S, K), K, Z_H) - q_S}{E + [D^{-1}(S(q_S, K), K, Z_H) - q_S] S(q_S)} = 0. \end{aligned} \quad (13)$$

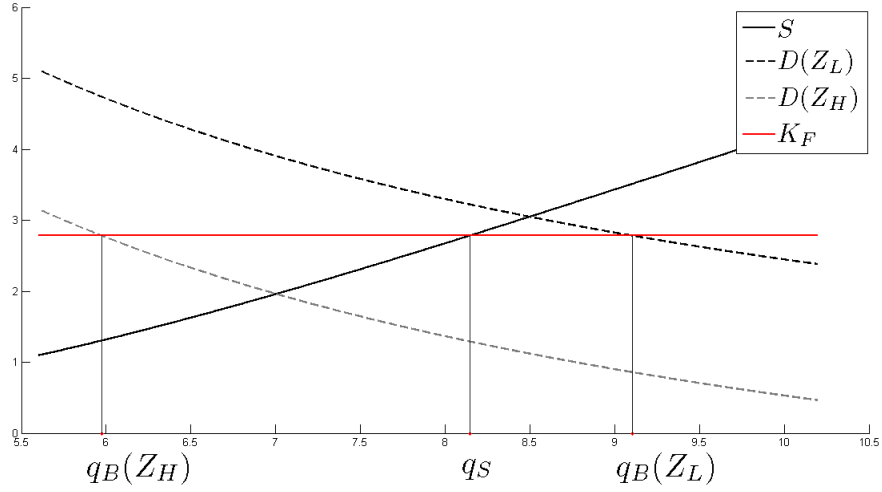
Or shortly:

$$\mathbb{E}_Z \left(\frac{D^{-1}(S(q_S, K), K, Z) - q_S}{E + [D^{-1}(S(q_S, K), K, Z) - q_S] S(q_S)} \right) = 0. \quad (14)$$

Observe that the LHS of 14 is a function of q_S and state variables E and K which are taken as given in the current period. This means that 13 can be used to pin down the equilibrium value of q_S . Given q_S we can calculate $I(q_S, K)$ and the next period value of aggregate capital: K' .

²⁰I use the fact that S^{-1} (with respect to first argument of function S) exists as S is strictly increasing and I apply similar arguments to D^{-1} .

Figure 5: Price formation in equilibrium



Once we have computed q_S , we are able to calculate k_F (from the market clearing condition $k_F = S(q_S, K)$). Given the current realization of Z , $q_B = D^{-1}(S(q_S, K), K, Z)$ we are able to pin down banker's wealth ω_F which together with Proposition 2 enables to compute E' . This means that it is possible to calculate the path of exogenous state variables analytically, without the need of using any global or any local solution methods. Price-formation in equilibrium is presented in Figure 5.²¹

It is clear that if the solution to 13 exists and is unique then given K , E , Z the values of K' and E' are well-defined (i.e., there is only one pair that is consistent with the equilibrium path). This in turn implies the existence and uniqueness of the RCE described in Definition 1.²²

Before I show the main result of this section, I prove the following auxiliary lemma:

Lemma 2. *If A1 holds then the following inequalities hold in equilibrium: $D^{-1}(S(q_S), K, Z_L) > q_S$, $D^{-1}(S(q_S), K, Z_H) < q_S$.*

I am in position to formulate the following theorem:

Theorem 1. *If A1 holds then solution to equation 13 exists and is unique.*

I finish this subsection with observation that characterizes the dependence of k_F on E .

Claim 1. *Aggregate reallocation of capital k_F increases with E .*

²¹Solid lines denote decisions/objects that result from choices made in the first stage of the period and dashed lines denote objects that are determined in the second stage.

²²This result follows because given the existence and uniqueness of q_S that solves 14 (given values $K \in \mathcal{K}$ and $E \in \mathcal{E}$, where \mathcal{K} and \mathcal{E} are spaces of state variables) we are able to compute $q_B(K, E, Z_L)$ and $q_B(K, E, Z_H)$. In other words, for all $K \in \mathcal{K}$ and $E \in \mathcal{E}$ the dynamic programming problem described by 9 and 10 is well-defined as we know the prices that are taken as given by the intermediary. It is therefore sufficient to apply the standard fixed-point argument (Banach theorem) to the dynamic programming problem characterized by 9 and 10 to argue that its solution exists and is unique.

Observe that Claim 1 gives rise to a direct link between condition of banks' balance sheets and the amount of capital reallocation in economy.

Transmission mechanism. Let us discuss the channels through which changes in Z affect the economy. Let us consider the situation at the end of the first stage, i.e., before the realization of Z . Observe that k_F is already chosen by banks and hence the value of deposits that needs to be repaid in the second stage - $q_S \cdot S(q_S, K)$ is fixed, too. Since q_S is defined in the first stage as well, then K' will remain unaffected by Z (see equation 8).

Suppose that the current realization of Z is $Z = Z_H$. By Lemma 2, this implies that $q_B(K, E, Z_H) < q_S$. Since k_F is already fixed, the value of ω_F drops (i.e., financial wealth of intermediaries falls). Since $e' = \beta\omega_F$ (by Proposition 2), then lower ω_F translates into decreased level of banks' equity in the next period. This in turn has adverse effects on the amount of intermediated capital k'_F (by Claim 1): k'_F decreases and the market clearing condition for "deposits" $k_F = S(q'_S)$ implies that q'_S falls. This has two effects: first, i-producers obtain less consumption goods $q'_S \cdot S(q'_S)$ that can be transformed in capital. Second, since q'_S is lower then $A_I^*(q'_S)$ grows and the proportion of i-producers that produce investment goods $\mathbb{P}(A_I \geq A_I^*(q'_S))$ falls. Both factors mean that I' is lower and hence the level of capital in the subsequent period K'' deteriorates which means that the aggregate output of consumption goods - i.e., $A_C (\pi_C K'')^\alpha L^{1-\alpha}$ is lower.²³ It is therefore clear that the condition of banks' balance sheets - e' is the only channel through which a decrease in demand for capital held by intermediaries (caused by $Z = Z_H$) affects the real economy.

The role of intermediaries. I finish this subsection with a comment on the role played by intermediaries. Notice that the decision about k_F is risky as it is made before the realization of Z .²⁴ This means that intermediaries absorb the risk that would be otherwise faced by capital sellers (i-entrepreneurs). More precisely, if banks were absent then capital sellers would sell capital to c-producers after the realization of Z and the price of this transaction would depend on the realization of aggregate uncertainty. If, however, banks are in place then i-entrepreneurs are insured against shifts in asset prices as they are offered riskless "deposit" contracts so that they purchase $q_S S(q_S)$ of consumption goods at price q_S that is independent of Z .

1.4 Monopolistically competitive intermediaries

In this section I study the economy in which intermediaries have a certain degree of monopoly power. I use a standard construction called the Dixit-Stiglitz aggregator which is applied in the market on which capital is sold to c-producers which gives rise to monopolistic competition in this market. Such formalization of intermediaries' monopoly power gives rise to their impact on prices (and quantities) of capital traded in economy and enables to calculate explicit formulas for banks' policy functions and do comparative statics exercises.²⁵ It is because the amount of channeled resources k_F remains a

²³It is easy to see that this aggregation result holds. It follows if one combines individual output $A_C k^\alpha l^{1-\alpha}$ with equations 4 and 12 and integrates this result over c-entrepreneurs.

²⁴Recall, that the choice of k_F is risky because it is assumed that banks do not have access to the storage technology of capital.

²⁵For instance, it is not possible to do qualitative exercises analogous to those conducted for the RCE with competitive banks if one considers a single monopolist instead of using the Dixit-Stiglitz aggregator.

linear function of equity e and thus the bank's budget constraint is linear in equity so that we can use the results shown by [Alvarez and Stokey (1998)] again. Following the literature, I do not introduce monopoly power in the market for "deposits".

Observe that considerations about the intermediaries' market structure do not affect producers' sector - this implies that entrepreneurs' policy functions, aggregate demand for capital $D(q_S, K, Z)$ and aggregate supply $S(q_S, K)$ of assets remain unchanged.

1.4.1 Capital retailers

To analyze the problem I introduce a new type of agent to the model: perfectly competitive retailers that earn zero profits, buy capital from monopolistic intermediaries and sell it to c-entrepreneurs. All these actions take place in the second stage.

In order to produce capital good k_F , a retailer must purchase a great many of wholesale capital goods $k_{F,j}$ indexed by $j \in [0, 1]$ (where j is an index assigned to a single banker). Retail good can be treated as a bundle/package of wholesale assets. Additionally, capital provided by intermediaries is differentiated and hence various capital goods are imperfect substitutes. This idea is formalized by the Dixit-Stiglitz aggregator:

$$k_F = \left[\int_0^1 k_{F,j}^{\frac{1}{\epsilon}} dj \right]^{\epsilon}, \quad \epsilon > 1, \quad (15)$$

where $\epsilon > 1$ measures the substitutability of different "pieces of capital" supplied by intermediaries. Profit function of the retailer reads:

$$q_B k_F - \int_0^1 q_{B,j} k_{F,j} dj. \quad (16)$$

Plugging 15 into 16 and deriving the FOC with respect to $k_{F,j}$ good yields:²⁶

$$q_B \left[\int_0^1 k_{F,j}^{\frac{1}{\epsilon}} dj \right]^{\epsilon-1} k_{F,j}^{\frac{1}{\epsilon}-1} = q_{B,j}.$$

I use 15 again to get the demand for capital of banker j :

$$k_{F,j} = \left(\frac{q_{B,j}}{q_B} \right)^{\frac{\epsilon}{1-\epsilon}} k_F. \quad (17)$$

Relationship described by 17 is taken as given by the monopolistic intermediary.

²⁶I am aware that from the point of view of measure theory, the derivative of both integrals is 0. It is because index j has measure zero so any change to "function" $k_{F,j}$ at point j has no effect on the integral. However, I use this formulation because it is common in the literature and it leads to the same FOC as: $k_F = \left(\sum_{j=1}^N k_{F,j}^{\frac{1}{\epsilon}} \right)^{\epsilon}$.

1.4.2 Monopolistic intermediaries

Bankers purchase capital in perfectly competitive market and sell it to retailers in a monopolistically competitive environment. In the first stage bank j solves:

$$W_1(e, K, E) = \max_{k_{F,j}} \mathbb{E}_Z (W_2(k_{F,j}, e, K, E, Z)), \quad (18)$$

and the second stage problem reads:

$$W_2(k_{F,j}, e, K, E, Z) = \max_{c, e'} \{ \log(c) + \beta W_1(e', K', E') \} \quad (19)$$

subject to :

$$c + e' = e + \left[q_B(K, E, Z) \cdot \left(\frac{k_F}{k_{F,j}} \right)^{1-\frac{1}{\epsilon}} - q_S(K, E) \right] k_{F,j},$$

$$E' = E'(K, E, Z),$$

$$K' = K'(K, E),$$

where a reformulated version of 17 - $q_{B,j} = q_B \cdot \left(\frac{k_F}{k_{F,j}} \right)^{1-\frac{1}{\epsilon}}$ has been plugged into the budget constraint. Let me concentrate on the symmetric case in which $k_F = k_{F,j}$. The following proposition characterizes policy functions of the monopolistic intermediary:

Proposition 3. *If A1 holds then decision rules and value function of the monopolistic intermediary are: $c = (1 - \beta)\omega_F$, $e' = \beta\omega_F$, $W_2 = \Psi^F(K, E, Z) + \frac{1}{1-\beta} \log \omega_F$, $k_{F,j} = \tilde{\Phi}(K, E) \cdot e$, where $\omega_F = e + (q_B - q_S) k_{F,j}$.*

Analytic form of $\tilde{\Phi}$ is presented in Appendix B.

1.4.3 Equilibrium

I do not present the full definition of Recursive Competitive Equilibrium with monopolistically competitive banks - it is analogous to the case of equilibrium with perfectly competitive intermediaries. Calculating K' and E' given K , E and Z requires analogous steps as in case of economy with competitive banking industry. Similarly to the previous case, equation that combines bank's FOC (with respect to k_F) and market clearing conditions plays a crucial role. The following formula is the equivalent of 20 in environment with monopolistically competitive banks:

$$\begin{aligned} \pi(Z_L) \cdot \frac{\frac{1}{\epsilon} D^{-1}(S(q_S(K, E)), K, Z_L) - q_S(K, E)}{e + [D^{-1}(S(q_S(K, E)), K, Z_L) - q_S(K, E)] S(q_S(K, E))} \\ \pi(Z_H) \cdot \frac{\frac{1}{\epsilon} D^{-1}(S(q_S(K, E)), K, Z_H) - q_S(K, E)}{e + [D^{-1}(S(q_S(K, E)), K, Z_H) - q_S(K, E)] S(q_S(K, E))} = 0. \end{aligned} \quad (20)$$

Observe, that the only difference between 13 and 20 is presence of fraction $\frac{1}{\epsilon}$ in 20. Under A1 I am able to prove the following result:

Theorem 2. *Under A1, solution to equation 20 exists and is unique.*

I finish this subsection with observation that characterizes the dependence of k_F on E :

Claim 2. In the RCE with monopolistically competitive intermediaries aggregate reallocation of capital k_F increases with E .

1.5 Comparison of economies with competitive and monopolistically competitive intermediaries

In this part I compare two economies - the one with competitive banks and the one monopolistically competitive intermediaries. It is instructive to divide the analysis into two subsections.

First, I show the potential advantages and disadvantages of the competitive banking sector in comparison to the monopolistic industry in the situation when state variables: K and E are the same in both economies. Second, I describe the long-run trade-off that is associated with the features of ergodic distributions of K and E .

1.5.1 The short-run trade-off

Competition and the amount of intermediated capital. Suppose that both economies have the same initial value of aggregate banks' equity - E and the same aggregate capital stock K . Next proposition characterizes the relationship between k_F^C and k_F^{MC} .

Proposition 4. *If the initial value of aggregate intermediaries' equity E and aggregate capital K are the same in both economies: the one with competitive banks and the one with monopolistically competitive intermediaries, then the amount of intermediated capital is strictly higher in economy with competitive banks.*

This result is illustrated in Figure 6.

Banks' losses in the crisis. Again, consider the situation when both economies have the same initial stock of banks' equity - E and aggregate capital K . Recall that in economy with competitive banks (by Lemma 2)²⁷:

$$q_B^C(Z_H) - q_S^C = D^{-1}(S(q_S^C), K, Z_H) - q_S^C < 0,$$

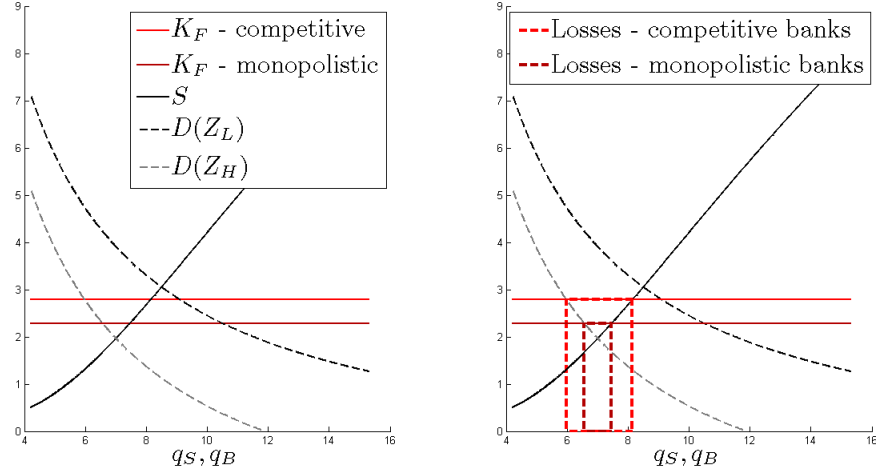
This means that losses incurred by competitive banks when $Z = Z_H$ are:

$$L^{CE}(Z_H) = (q_B^C(Z_H) - q_S^C) \cdot S(q_S^C) < 0. \quad (21)$$

Let us compare 21 with losses generated by monopolistic intermediaries. There are two effects that magnify the losses of competitive industry in comparison to monopolistically competitive bankers.

²⁷I suppress the dependence of pricing functions q_B and q_S on K and E for notational convenience.

Figure 6: Capital intermediated in economy with monopolistically and in economy with competitive banks



First, by Proposition 4 and market clearing condition for "deposits" we get $S(q_S^C) > S(q_S^{MC})$ and hence:

$$q_B^C(Z_H) = D^{-1}(S(q_S^C), K, Z_H) < D^{-1}(S(q_S^{MC}), K, Z_H) = q_B^{MC}(Z_H).$$

This together with the fact that $q_S^C > q_S^{MC}$ implies:

$$q_B^C(Z_H) - q_S^C < q_B^{MC}(Z_H) - q_S^{MC} < 0. \quad (22)$$

Inequality 22 means that one reason for which competitive intermediaries generate higher losses than monopolistic banks is due to the fact that they do not internalize the influence of their portfolio decisions (i.e., the decision about k_F^C) on prices.

Second, since $S(q_S^C) > S(q_S^{MC})$ the uninternalized effect on prices is amplified even further which means that:

$$L^C(Z_H) = (q_B^C(Z_H) - q_S^C) S(q_S^C) < (q_B^{MC}(Z_H) - q_S^{MC}) S(q_S^{MC}) = L^{MC}(Z_H).$$

These considerations are summarized by the following proposition:

Proposition 5. *If the initial value of aggregate intermediaries' equity E and the capital stock K are the same in both economies: the one with competitive banks and the one with monopolistically competitive intermediaries, then aggregate losses generated by banks for $Z = Z_H$ (i.e. "crisis") are higher in economy with competitive intermediaries.*

Proposition 5 has an important dynamic consequence: if $Z = Z_H$ occurs in the initial period then $\omega_F^{MC} > \omega_F^C$ and hence monopolistically competitive banks accumulate higher equity E' . This coupled with results presented in Propositions 1 and 2 means that the amount of capital transferred

from i-producers to c-entrepreneurs in the subsequent period can be strictly lower for the economy with competitive banks than in economy with monopolistically competitive intermediaries.²⁸ These considerations are shown in Figure 6.

1.5.2 The long-run trade-off

In this subsection I analyze ergodic distributions of K and E under two different regimes (perfectly competitive and monopolistically competitive banks). First, I present analytic characterization of the upper and lower bounds of the support of ergodic densities. Second, I use numerical simulations to explore some additional features of those distributions that are tightly associated with the results concerning the bounds. First of all, however, let us modify the model to make the analysis more tractable. In particular, to simplify the exposition I assume that $\mathbb{P}(A_I = 1) = 1$, i.e. all i-producers have the same level of productivity.²⁹ This assumption holds throughout this section and Sections 1.6 and 1.7.

To guarantee that equilibrium with $\mathbb{P}(A_I = 1) = 1$ exists, I assume that parameters satisfy the following inequality:

$$\left[\frac{(1 + \phi Z_L) \frac{\pi_L}{\pi_C} + \phi Z_L}{(1 + \phi Z_H) \frac{\pi_I}{\pi_C} + \phi Z_H} \right] \frac{1 + \phi}{\pi_I} > \frac{1}{\delta} - 1. \quad (23)$$

It is easy to see that the set of parameters which satisfies 23 is non-empty - it is because the LHS of 23 is always strictly positive and the limit of the RHS when $\delta \rightarrow 1$ is zero. First, notice that the necessary condition for existence of equilibrium is:

$$\forall_K \frac{G(K)}{(1 + \phi Z_H) \frac{\pi_I K}{\pi_C} + \phi Z_H} > 1 - \delta. \quad (24)$$

The LHS of 24 is the inverse demand function evaluated at $\pi_I K$ (amount of capital supplied by i-entrepreneurs when $q_S(K, E) > 1 - \delta$). Condition 24 says that the aggregate demand curve for capital channeled by banks (that corresponds to realization Z_H of the aggregate shock) intersects the $S(q_S, K)$ scheme for such value of q_S that $S(q_S, K) = \pi_I \cdot K > 0$. It is because I want to exclude the situation in which $D(q_B, K, Z_H)$ and $S(q_S, K)$ cross each other at $q_B = q_S < 1 - \delta$ if $Z = Z_H$ (which would imply that the supply of capital is 0). The following lemma shows that 24 is true when condition 23 is satisfied:

Lemma 3. *Condition 24 holds if parameters satisfy 23.*

The economy with $\mathbb{P}(A_I = 1) = 1$ is described in Appendix A in a more detailed manner. It is easy to extend those result to describe the model with $\mathbb{P}(A_I = 1) = 1$ and $\epsilon > 1$.

Let us start with the lower bounds on ergodic densities of K^C , K^{MC} , E^C and E^{MC} (these variables denote aggregate capital in economy with competitive banks, aggregate capital in economy with monopolistically competitive banks, aggregate equity in economy with competitive banks, aggregate

²⁸Observe that it may not be the case due to the monopolistic friction.

²⁹It can be shown numerically that the analytical results presented in this section continue to hold for the non-degenerate distribution of A_I .

equity in economy with monopolistically competitive banks, respectively).³⁰ It is easy to show that the following proposition holds:

Proposition 6. *The common lower bound on the supports of ergodic densities associated with E^C and E^{MC} is $\underline{E} = 0$.*

To obtain this result I have used the Borel-Cantelli lemma and the law of motion for E . The next proposition, that characterizes the lower bounds for K^C , K^{MC} , requires some more refined arguments than those used in the proof of Proposition 6:

Proposition 7. *If $\mathbb{P}(A_I = 1) = 1$ and condition 23 hold then the common lower bound on the supports of ergodic densities associated with K^C and K^{MC} is $\underline{K} = \left(\frac{\Psi}{\delta}\right)^{\frac{1}{1-\alpha}}$ where Ψ is a function of parameters.*

One remark is in order. Since the probability of the crisis $\pi(Z_H)$ is significantly lower than the probability of a "good" shock $\pi(Z_L)$ then the chance that the aggregate level of capital approaches to \underline{K} is extremely low.³¹ This in turn means that the value of \underline{K} has a negligible influence on the moments associated with ergodic distributions of K^C and K^{MC} . It is therefore much more important to study the upper bounds on K and E . The next proposition establishes the relationship between the upper bounds on K^C and K^{MC} (let us denote them by \bar{K}^C and \bar{K}^{MC}):

Proposition 8. *If $\mathbb{P}(A_I = 1) = 1$ and condition 23 hold then $\frac{d\bar{K}^{MC}}{d\epsilon}$ evaluated at $\epsilon = 1$ is negative.*

Proposition 8 says that the upper bound of the long-run distribution of capital decreases when perfectly competitive market becomes monopolistic. On the one hand it is intuitive because when $\epsilon > 1$ then intermediaries increase their profits and less resources (consumption goods) is transferred to investors that create new capital. On the other hand, one could argue that this effect can be mitigated (or even eliminated) because if banks have higher profits then their long-run equity should be, too (this intuition is confirmed by Proposition 9). This in turn, together with Claim 2, could imply that the negative effect of the growth in ϵ could be outweighed by the impact of higher equity (see **Hypothesis 2** presented in the Introduction). Proposition 8 states that this potentially mitigating effect is too weak and hence \bar{K}^{MC} decreases in ϵ . Since the $\pi(Z_L)$ is significantly larger than $\pi(Z_H)$ then the value of upper bounds of supports of ergodic densities will affect the moments of ergodic distributions.

The next proposition describes the impact of ϵ on \bar{E}^{MC} . To prove this statement it is sufficient to assume one additional requirement, i.e. that $\pi(Z_L)\beta > \alpha$ holds:³²

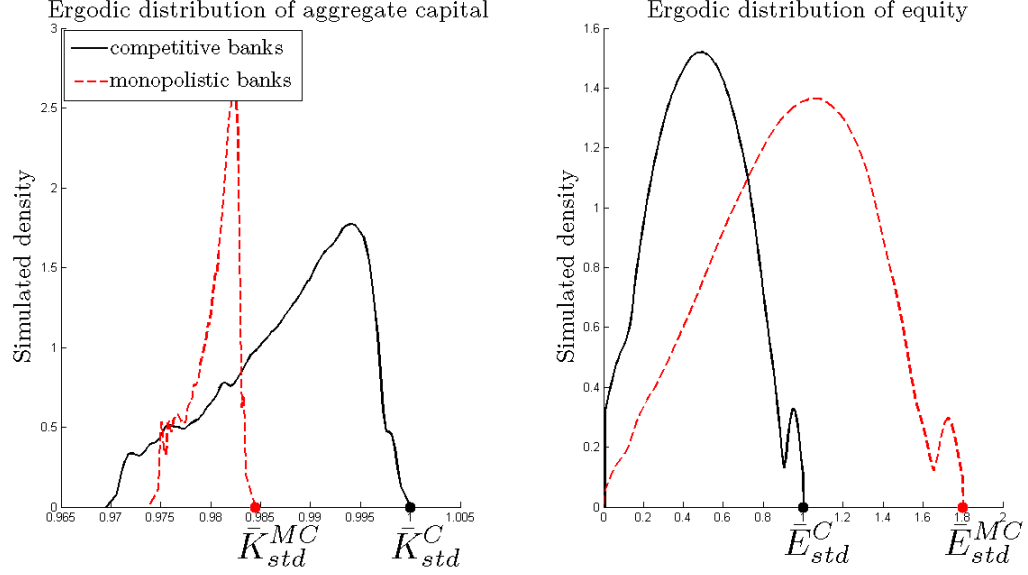
Proposition 9. *If $\pi(Z_L)\beta > \alpha$, $\mathbb{P}(A_I = 1) = 1$ and condition 23 hold then $\frac{d\bar{E}^{MC}}{d\epsilon}$ evaluated at $\epsilon = 1$ is positive.*

³⁰Observe that I assume the existence of ergodic densities. If they do not exist (see for example the Radon-Nikodym theorem) then all results in this section can be reformulated in terms of probability measures which is always possible.

³¹Simulations show that the number of consecutive realizations of $Z = Z_H$ required for the economy to find itself in the close neighborhood of the level $\underline{K} = \left(\frac{\Psi}{\delta}\right)^{\frac{1}{1-\alpha}}$ is ≈ 500 .

³²Observe that since Z_H is a rare event then $\pi(Z_L)$ is close to 1, the same is true for β . Since in the RBC literature it is assumed that $\alpha \approx 0.33$ then this additional condition is not very restrictive.

Figure 7: Ergodic distributions



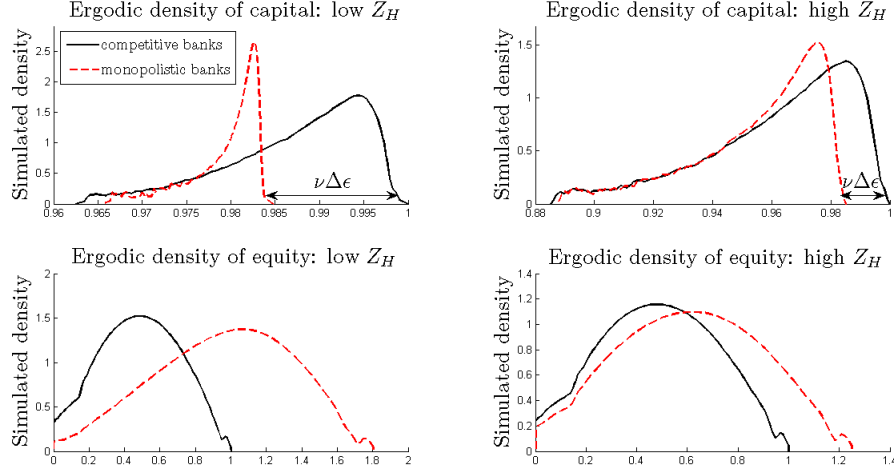
Again, Proposition 9 shows which of the two forces affecting \bar{E}^{MC} is stronger when ϵ increases. The first force increases banks' profits when ϵ grows because intermediaries have a stronger impact on prices q_B^{MC} . The second effect implies that if ϵ increases then (by Proposition 8) average K^{MC} drops and hence the amount of intermediated capital k_F^{MC} is lower. This affects intermediaries' profits in a negative way as $(q_B^{MC}(Z) - q_S^{MC}) k_F^{MC}$ (see **Hypothesis 1** presented in the Introduction). Proposition 9 shows that the latter effect is dominated by the first one. This in turn means that monopolistically competitive industry accumulates higher equity buffer against adverse aggregate shocks.

To illustrate the consequences of Propositions 6-9 let us use numerical simulations. Results are shown in Figure 7. I standardize the values of aggregate variables: aggregate capital is divided by the upper bound \bar{K}^C and aggregate equity is divided by \bar{E}^C .³³ Simulation confirms the results presented in Propositions 8 and 9. The upper bound on K is higher in the economy with perfectly competitive banks and the upper bound on E is higher for the economy with monopolistically competitive banks. As it has been expected, ergodic densities exhibit a significant concentration in the neighborhood of the upper bounds since $\pi(Z_H) < \pi(Z_L)$.

Observe that the fact that $\bar{E}^C < \bar{E}^{MC}$ (this relationship is certainly inherited by the means of ergodic distributions) has an additional, important consequence. Since the aggregate equity of banks tends to be higher in the economy with monopolistic intermediaries then the financial system has greater capacity to absorb adverse shocks Z_H . Hence, not only is the variance of K (and hence the variance of output) significantly lower in the economy with monopolistic banks but also recessions experienced by the economy with perfectly competitive banks are more severe. This property is in

³³Standardized values of the upper bounds are denoted by: $\bar{K}_{std}^C = \frac{\bar{K}^C}{\bar{K}^C} = 1$, $\bar{K}_{std}^{MC} = \frac{\bar{K}^{MC}}{\bar{K}^C}$, $\bar{E}_{std}^C = \frac{\bar{E}^C}{\bar{E}^C} = 1$, $\bar{E}_{std}^{MC} = \frac{\bar{E}^{MC}}{\bar{E}^C}$.

Figure 8: Market structure and aggregate risk



line with evidence presented in Figure 1 and with considerations of [Gorton (2010)] concerning the relationship between competition in the financial sector and macroeconomic stability.

1.6 Market structure of the financial sector and aggregate risk

In this section I study the impact of changes in aggregate risk on performance of different market structures of the banking sector. Similarly to the previous section I concentrate on the upper bounds of supports of ergodic densities. In what follows I study the impact of changes in the magnitude of adverse shock Z_H on relative difference in performance between competitive financial industry and monopolistic banking sector. Proposition 10 shows that changes in aggregate uncertainty (in the value of Z_H) have no impact on the standardized difference in behavior between economy with competitive banks and economy with monopolistic intermediaries in the neighborhood of the upper bound \bar{K}^C :

Proposition 10. *If $\pi(Z_L)\beta > \alpha$, $\mathbb{P}(A_I = 1) = 1$ and condition 23 hold then $\left(\frac{d\bar{K}^{MC}}{d\epsilon}\right)$ (evaluated at $\epsilon = 1$) does not depend on Z_H and its value is a function of parameters different from Z_H (denoted by $\nu > 0$).*

I consider a standardized value of derivative $\frac{d\bar{K}^{MC}}{d\epsilon}$ because changes in Z_H affect the value of the upper bound \bar{K}^C (or \bar{K}^{MC}). The next proposition analyzes the impact in changes in aggregate uncertainty on the difference in equity accumulation between the regime with competitive and with monopolistic banks:

Proposition 11. *If $\pi(Z_L)\beta > \alpha$, $\mathbb{P}(A_I = 1) = 1$ and condition 23 hold then $\left(\frac{d\bar{E}^{MC}}{d\epsilon}\right)$ (evaluated at $\epsilon = 1$) decreases in Z_H .*

Proposition 11 shows that changes in aggregate risk do affect the difference in accumulation of equity between competitive and monopolistic banks. More precisely, if financial disturbances are larger (i.e. Z_H increases) then the difference in ergodic distribution of equity buffer between two market structures declines.

Let me interpret the results described by Propositions 10 and 11. It will be instructive to simulate the model for the low value of Z_H and high value of Z_H and compare ergodic distributions for those two cases. Figure 8 shows the results. Notice that simulation confirms the finding described in Proposition 11: the standardized upper bounds for ergodic density of E when Z_H is low (the bottom left panel) are more distant from each other than those in the bottom right panel of Figure 8. In general, it can be observed that both distributions "converge" to each other as Z_H grows. This happens because banks' behavior exhibits an increase in precautionary motives as Z_H rises: intermediaries' risk exposure grows (as the realized losses can be potentially higher) and hence they decide to channel less funds. This "precautionary component" of banks' behavior is the same for both monopolistic and competitive banks. In case of monopolistic banks there is additional important motive that influences their behavior since they exercise their market power. The relative role of this "monopolistic" component declines as Z_H grows (i.e., as the "precautionary component" expands). This is why ergodic distributions for monopolistic and competitive banks become more similar as Z_H increases.

Now, let me point two additional remarks out. First, notice that the distance between the standardized upper bounds on capital for the competitive and monopolistic regimes is the same in the top panels in Figure 8.³⁴ This implies that the negative impact of the monopolistic wedge on capital accumulation is persistent and it remains unaffected by changes in Z_H . Second, observe that (quite surprisingly) the economy with competitive banks does not exhibit much more severe recessions than the monopolistic regime if aggregate adverse shocks become large. It is somewhat counterintuitive as one could expect that competitive banks should not be able to absorb large losses during financial crises. This reasoning ignores the "precautionary" mechanism described above: as the magnitude of Z_H increases, competitive banks decide to channel less funds. This "precautionary" behavior makes them similar to monopolistic intermediaries and hence the severity of recessions under both regimes is very much alike.

1.7 Inefficiency of the RCE with competitive banks

In this section I show that the decentralized allocation in economy with competitive banks is inefficient. To simplify the exposition condition $\mathbb{P}(A_I = 1) = 1$ continues to hold. I do that to be able to formulate the planner's problem in a tractable way.³⁵ First, I point out an important feature of the

³⁴Notice that $\Delta\epsilon$ is defined as: $\Delta\epsilon \equiv \epsilon - 1$.

³⁵More precisely: if I assume that f (the pdf associated with random variable A_I) exists and has support of a strictly positive measure (not necessarily unbounded) then the social planner should be able to transfer all consumption goods he wants to transform into investment to the i-producer with highest productivity level A_I^* (see [Kurlat (2013)]). This gives rise to additional source of inefficiency of the RCE, as in case of competitive equilibrium there are i-producers with productivity level strictly lower than A_I^* that sell their entire capital and transform some part of consumption goods they purchase into capital (there are such producers because in the RCE $I > 0$ - there is no trade in assets otherwise; if, by contradiction, only i-producers with A_I^* invest then $I = 0$ as point A_I^* has measure zero for the measure associated with f). I would like to isolate my analysis from this inefficiency so I assume that $\mathbb{P}(A_I = 1) = 1$.

RCE allocation with competitive banks. Second, I formulate the social planner's problem and I solve it. Finally, I compare both allocations and I identify sources of differences between them. In other words I investigate the reasons for which the allocation associated with the RCE is inefficient. To shorten the exposition derivations are postponed to Appendix A.

Decentralized solution. As we shall see, the main difference between the decentralized solution and the optimal outcome is the dependence of capital accumulation process on aggregate shocks Z . Let us therefore derive the formula for aggregate investment in the decentralized economy in which $\mathbb{P}(A_I = 1) = 1$ (it is assumed that condition 23 holds). First, by Proposition 1, we conclude that individual investment satisfy:

$$i = k' = \frac{q_S k}{1 + \phi}. \quad (25)$$

Since the supply of capital in case for which $\mathbb{P}(A_I = 1) = 1$ is $\pi_I K$ (see Appendix A) then formula for aggregate investment reads:

$$I(q_S, K) = \frac{q_S(E, K)}{1 + \phi} \cdot \pi_I \cdot K. \quad (26)$$

Observe that from $I_{q_S} > 0$ and by Claim that can be found in Appendix A we can conclude that I varies with aggregate level of banks' equity E . This means that aggregate shock Z (that influences E) has an impact on aggregate investment and capital accumulation.

Efficient allocation. Let us analyze the problem that is solved by the benevolent social planner that attaches equal Pareto weights to all agents. Planner chooses investment and consumption of i-producers, c-producers, workers and financial intermediaries subject to the resource constraint:

$$\pi_C c_C + \pi_I c_I + c_F + L c_L + I = A_C K^\alpha L^{1-\alpha}$$

and subject to the law of motion for capital:

$$K' = (1 - \delta)K + I. \quad (27)$$

Moreover, planner faces the same informational frictions as individual agents in the RCE: he makes decisions about c_I, c_L, K' and I before the realization of the preference shock Z . Hence his maximization problem can be summarized by the system of two Bellman equations (I have used 27 to eliminate I):

$$P_1(K) = \max_{c_I, c_L, K'} \mathbb{E}_Z (P_2(K, K', Z)),$$

$$P_2(K, K', Z) = \max_{c_C, c_F} \{ \pi_C \cdot Z \cdot \log c_C + \pi_I \log c_I + \log c_F + L \log c_L + \beta P_1(K') \}.$$

subject to :

$$\pi_C c_C + \pi_I c_I + c_F + L c_L + K' - (1 - \delta)K = A_C K^\alpha L^{1-\alpha}$$

where P_1 and P_2 are value functions associated with planner's problem. Derivation of the solution to

planner's problem is shown in Appendix A. It is characterized by the following equation:

$$\beta P'_1(K') = \frac{1 + \mathbb{E}_Z Z \cdot \pi_C + L + \pi_I}{A_C K^\alpha L^{1-\alpha} - K' + (1-\delta)K}.$$

Notice that the equation above defines K' as an implicit function of K . This (together with the law of motion for capital) implies that I is function of K and hence it is not affected by the past realizations of Z . This fact makes it very different from the aggregate investment under the RCE with competitive banks: planner's solution implies that capital K follows a deterministic path which is independent of shocks to Z .

Except for the dissimilarity in aggregate investment levels there is an additional, significant difference between the optimal outcome and the allocation associated with the RCE. Notice, that idiosyncratic shocks (both to A_I and those associated with investment opportunities) and the fact that entrepreneurs have only one instrument to smooth consumption (capital holdings) leads to a nondegenerate distribution of asset holdings across producers. This coupled with Proposition 1 means that producers of the same type (i.e., either c-entrepreneurs or i-entrepreneurs) have different consumption levels. This result differs from the planner solution that assigns such consumption plans that each category of entrepreneurs has the same consumption level. In other words, planner decides to insure producers against the idiosyncratic shocks. The only difference between their consumption levels (under the efficient solution) results from the redistributive behavior of the planner that reacts to changes in Z and decides to transfer more goods to c-producers at the cost of lower consumption of financial intermediaries.

Sources of inefficiency of the RCE allocation are discussed in Appendix A in a more detailed way. One comment is in order here. Observe that the fact that aggregate capital follows a deterministic path in planner's solution implies that its ergodic distribution is a mass point. This in turn means that elimination of fluctuations in output is socially desirable. As we have seen in Section 1.5.2, the presence of the monopolistic banking sector dampens aggregate fluctuations which could suggest that it is welfare-improving in comparison to the competitive regime. This argument, however, ignores the fact that the level of output tends to be higher when banks are competitive (Proposition 8). It is therefore essential to ask what is the socially optimal level of output (or equivalently - capital) and how it is related to the decentralized outcome?³⁶ The following inequality provides a condition under which the optimal level of capital is higher than the upper bound for ergodic distribution of capital in economy with competitive banks:

$$K_{opt} = L \left(\frac{\alpha A_C}{\frac{1}{\beta} - 1 + \delta} \right)^{\frac{1}{1-\alpha}} > \frac{L}{\pi_C} \left[\frac{\left\{ \pi(Z_L) \frac{\beta}{1-\beta} \frac{1}{\kappa(Z_L)} + \left(\pi(Z_L) + \frac{\pi(Z_H)}{1-\beta} \right) \frac{1}{\kappa(Z_H)} \right\} \alpha A_C}{\frac{1}{1-\beta} \frac{\delta(1+\phi)}{\pi_I}} \right]^{\frac{1}{1-\alpha}} = \bar{K}^C$$

where $\kappa(Z) = (1 + \phi Z) \frac{\pi_I}{\pi_C} + \phi Z$. It can be checked that if π_C is close to 1 or in the neighborhood of 0 then the inequality above holds (the latter case can be ignored since it is assumed that 23 is satisfied).³⁷

This in turn means that if the idiosyncratic risk is sufficiently low (i.e., if the chance that entrepreneur

³⁶This question is not trivial because entrepreneurs that accumulate capital face an analogous problem to agents in the setting introduced by [Aiyagari (1994)]: they make their decisions facing incomplete markets (their only instrument to insure is stock of capital), borrowing constraints (capital holdings cannot be negative) and idiosyncratic risk (lack of opportunity to produce consumption goods). This may lead to "overaccumulation" of capital which is not socially desirable.

³⁷The easiest way to verify this statement is to start with case in which there is no aggregate uncertainty: $Z_H = Z_L$.

is able to generate consumption goods is high) then the overaccumulation of capital does not happen and hence the "level vs. volatilities" trade-off between competitive and monopolistic regimes is likely to occur.³⁸

1.8 Conclusions

I have presented a tractable dynamic general equilibrium model with financial sector that was applied to study the business cycle consequences of changes in competition in the financial sector. I have used the model to investigate the dynamic properties of two regimes: the one with competitive banks and the second with monopolistically competitive intermediaries.

More precisely, I have concentrated on two time horizons: the short-run perspective and the long-run perspective. The first one indicated that competitive banking industry guarantees higher level of intermediation activities but at the same time it exhibits higher exposure to aggregate risk (losses generated by competitive banks are larger than those incurred by monopolistic intermediaries). Therefore if an adverse aggregate shock arrives, equity of competitive banks is drained more severely which impedes intermediation in subsequent periods. This in turn means that negative impact of monopolistic wedge on the amount of channeled funds can be outweighed by greater intermediation ability of monopolistic banks during economic downturns.

The long-run perspective concerned the analysis of ergodic distributions of aggregate variables. In particular, I have shown that the short-run trade-off has its counterpart in the long-run: on the one hand ergodic density of capital (and output) under competitive regime has its upper bound shifted to the right in comparison to the upper bound of density associated with monopolistic regime. The opposite relationship is true for the upper bounds of ergodic densities of banks' equity. This has an important consequence: higher equity cushion of monopolistic banks cushions adverse aggregate shocks more effectively which in turn implies lower aggregate uncertainty induced by monopolistic financial sector. Moreover, I have studied the impact of changes in magnitude of preference shocks on both market structures. Surprisingly, the presence of larger "bad" shocks does not deteriorate the performance of competitive regime in comparison to monopolistic one. This happens because intermediaries response with more precautionary behavior to increases in the size of "bad" shocks which makes behavior of banks under two regimes very much alike. Therefore, the ability of competitive banks to absorb aggregate shocks is similar to the one exhibited by monopolistic financial institutions.

Last, I characterize the planner's solution and discuss its relationship to economies with monopolistic and competitive intermediaries. Optimal outcome exhibits no aggregate fluctuations in the long run. This qualitative feature makes it similar to economy with monopolistic banks. This, however, does not mean that monopolistic market structure outperforms competitive one in terms of welfare - it is because economy with competitive banks tends to have higher output level than the one with

³⁸Observe that I have not addressed the issue of welfare under various market structures in a direct way (i.e., by computing the value of welfare criterion that aggregates individual utilities). It is because there are three types of agents in my model and a discretionary choice of Pareto weights could affect the outcome of such an exercise substantially (e.g., high Pareto weights attached to bankers would make the monopolistic regime more socially-desirable in comparison to the competitive one).

monopolistic intermediaries.

Appendix A

Discussion about the assumptions

Let us come back to the model in which the distribution of productivity A_I is non-trivial. In this part, I discuss the key assumptions that has been made so far.

Independent and identically distributed aggregate shock. This assumption is made for three reasons.

First, I make it because I want to eliminate the influence of shocks' persistence on agents' decisions. In particular, if I assumed that Z is Markovian then $\pi(Z_L)$ and $\pi(Z_H)$ would be replaced by $\pi(Z_L|Z_{-1})$ and $\pi(Z_H|Z_{-1})$, respectively where $\pi(\cdot|Z_{-1})$ is probability measure of current aggregate shock conditional on the previous realization of Z - Z_{-1} . Then it would imply that q_S (and by market clearing conditions k_F , too) that is implicitly defined by 13 depends not only on E but also on Z_{-1} . Hence it would be hard to isolate the influence of E on k_F from the impact of agents' expectations about the realization of Z (captured by $\pi(Z_L|Z_{-1})$ and $\pi(Z_H|Z_{-1})$) on banks' decision about k_F . Since the former is the key force in my analysis and it is a channel that is significantly affected by changes in the intermediaries' market structure then I wanted to keep it clear and isolated from influence of any additional factors.

Second, if despite the assumption about i.i.d. shocks, the model is able to generate persistent changes in economic aggregates then importance of the underlying acceleration mechanism (that works through the effect of E on k_F and Y in my model) is shown. A similar argument for using i.i.d. shocks is presented in [Bernanke and Gertler (1989)].

Third, this assumption enables me to calculate the closed-form solutions for the value function and the associated policies of producers (i.e., functions presented in Proposition 1).

In Section ?? I add Markovian productivity shock to the model which makes it impossible to solve the model analytically.

Non-degenerate distribution of productivity A_I . Observe that if a continuous density f (with support \mathbb{R}_+) characterizes the distribution of A_I then supply of capital $S(q_S, K)$ is an increasing and differentiable function of q_S with $S(0, K) > 0$. This implies that we do not need to make any additional assumptions about parameters (analogous to condition 23) to guarantee the existence of RCE. This in turn means that we do not impose any additional constraints on parameters that could constrain parametrization/calibrations of the model. Moreover, this assumption gives rise to an additional channel through which price q_S (and conditions of banks' balance sheets) affects the real economy (in particular, the aggregate investment). This channel changes the extensive margin of investment since q_S affects the investment decisions of i-producers. For instance, if q_S jumps then more i-entrepreneurs find their investment opportunities profitable and hence more producers sell their entire capital to finance their investment project. This mechanism is absent if we consider the model with equal investment opportunities (e.g., $\mathbb{P}(A_I = 1) = 1$).

C-producers and i-producers that switch their types over time. Similarly to [Bigio (2015b)] I use a random and i.i.d. assignment of producer types. The randomness reduces the state space: if it

is relaxed then we would have to keep track of both capital held by i-producers and c-producers. Assumption about the i.i.d. structure of these shocks could be replaced by the Markovian setup in which distribution of entrepreneurs across the two types is stationary of the corresponding Markov chain.³⁹ This would make the notation more complex and worsen the clarity of exposition. Since replacing the assumption about i.i.d. assignments by Markovian ones would keep the qualitative features of my results unaffected then I follow the simpler stochastic structure in this work.

Different production technologies. Observe that there are two production technologies: a linear one (given by formula $A_I \cdot i$) and the Cobb-Douglas technology that is operated by c-entrepreneurs (that uses two inputs: capital and labor). I assume this asymmetry (i.e., that investment goods are not produced by means of the Cobb-Douglas technology) to create a channel through which the amount of intermediation affects real economy. Observe that if investment goods are produced directly from consumption goods transferred by banks then this channel emerges in a natural way: the more capital k_F is transferred by banks from i-entrepreneurs to c-producers, the higher is the amount of resources (consumption goods) that can be used for production of new capital by i-producers. It is because q_S increases together with k_F and hence $q_S \cdot k_F$ grows as well.

Derivations from Section 1.5.2

Let us describe how the economy with $\mathbb{P}(A_I = 1) = 1$ looks like. I use Lemma 1 to conclude that all i-entrepreneurs invest only if:

$$q_S(K, E) \geq 1 - \delta.$$

If this condition does not hold the none of them invest. This implies that the capital supply function takes the following form:

$$S(q_S, K) = \begin{cases} \pi_I \cdot K & \text{if } q_S(K, E) \geq 1 - \delta \\ 0 & \text{otherwise} \end{cases} \quad (28)$$

Observe that 28 and the market clearing for "deposits" imply that the amount of intermediated capital is not dependent on E . The problem of c-producer remains unchanged so aggregate demand for capital is:

$$D(q_B, K, Z) = \left[\frac{1}{1 + \phi Z} \cdot \frac{G(K)}{q_B} + \frac{1}{1 + \phi Z} - 1 \right] \cdot \pi_C \cdot K.$$

Since $S(q_S, K)$ is not a continuous function then we need an additional argument to show that equilibrium exists if $\mathbb{P}(A_I = 1) = 1$. This condition is shown in the main text, and is summarized by Lemma 3.

Intermediaries solve the same problem as before. We are in position to prove existence and uniqueness of equilibrium in the simplified environment. Similarly to the more general case the equilibrium condition (i.e. bank's FOC combined with market clearing for deposits and capital sold to c-entrepreneurs) plays crucial role (recall that if 23 holds then $S(q_S, K) = S(K) = \pi_I K$; this implies, additionally, that inverse demand function D^{-1} is independent of E):

³⁹This assumption would be more realistic because producers would switch their types in a persistent manner.

$$\begin{aligned} \pi(Z_L) \cdot \frac{D^{-1}(K, Z_L) - q_S}{E + [D^{-1}(K, Z_L) - q_S] \pi_I K} \\ \pi(Z_H) \cdot \frac{D^{-1}(K, Z_H) - q_S}{E + [D^{-1}(K, Z_H) - q_S] \pi_I K} = 0. \end{aligned} \quad (29)$$

The following theorem shows that q_S that solves 29 exists and is unique.

Theorem. *If 23 holds then solution to equation 29 exists and is unique.*

It is clear that the amount of reallocated capital is independent of E as it is always equal to $\pi_I K$. This means that result analogous to Claim 1 does not hold. It does not mean however that q_S is not related to changes in E . This relationship is summarized by the following claim:

Claim. Price q_S paid by banks for capital bought from i-producers increases in E (for K kept constant).

Inefficiency of the RCE with competitive banks

Solution to the planner's problem. Let us compute the solution to the second stage problem (it is derived for given values of c_I , c_L , K' and K). Let us define the amount of resources available during the second stage:

$$\Omega(K, K', c_I, c_L) = A_C K^\alpha L^{1-\alpha} - \pi_I c_I - L c_L - K' + (1 - \delta)K. \quad (30)$$

Combining the FOCs associated with c_C and c_F yields:

$$c_C = Z c_F.$$

Plugging into the resource constraint yields:

$$\begin{aligned} c_F &= \frac{\Omega(K, K', c_I, c_L)}{1 + Z \pi_C}, \\ c_C &= \frac{Z \cdot \Omega(K, K', c_I, c_L)}{1 + Z \pi_C}. \end{aligned}$$

We use these results to reformulate the first stage problem:

$$\begin{aligned} P_1(K) &= \max_{c_I, c_L, K'} \mathbb{E}_Z \left(\pi_C \cdot Z \cdot \log \left(\frac{Z \cdot \Omega(K, K', c_I, c_L)}{1 + Z \pi_C} \right) + \pi_I \log c_I \right. \\ &\quad \left. + \log \left(\frac{\Omega(K, K', c_I, c_L)}{1 + Z \pi_C} \right) + L \log c_L + \beta P_1(K') \right). \end{aligned}$$

Since we have log preferences we can extract terms $\frac{Z}{1+Z\pi_C}$ and $\frac{1}{1+Z\pi_C}$ which simplifies our further calculations. FOCs associated with per capita consumption levels c_I and c_L yield:

$$c_I = c_L = \frac{\Omega(K, K', c_I, c_L)}{1 + \mathbb{E}_Z Z \cdot \pi_C}.$$

First order condition for K' is:

$$\beta P'_1(K') = \frac{1 + \mathbb{E}_Z Z \cdot \pi_C}{\Omega(K, K', c_I(K), c_L(K))}. \quad (31)$$

Let us plug formulas for c_I and c_L into 30 and then combine it with 31 to get:

$$\beta P'_1(K') = \frac{1 + \mathbb{E}_Z Z \cdot \pi_C + L + \pi_I}{A_C K^\alpha L^{1-\alpha} - K' + (1 - \delta)K}. \quad (32)$$

Sources of inefficiency of the RCE allocation. Observe that producers cannot fully insure against the next period's value of idiosyncratic shock - they can use either "deposits" (if they are i-entrepreneurs) or purchase capital from intermediaries (if they are c-producers) but none of these options can insure them against being i-producer, insure them against becoming c-producer next period and simultaneously protect them against shifts in Z .⁴⁰ Incompleteness of insurance markets faced by producers leads to a non-degenerate distribution of capital holdings and different consumption levels across entrepreneurs of the same type - this allocation feature is absent in case of the planner solution.

Incompleteness of insurance markets faced by intermediaries means that they cannot reduce the aggregate risk associated with shifts in demand for assets caused by changes in Z . Observe that if this risk is eliminated (e.g., by transfers that cover potential losses if the difference between the value of assets sold and deposits that has to be repaid is negative) then price q_S would move towards $q_B(Z_L)$. The latter price, by the previous discussion, does not depend on E and hence both the value of reallocated capital and aggregate investment becomes independent of history Z which establishes a qualitative similarity between the planner's solution and the RCE with transfers on the aggregate level. Hence the market incompleteness faced by banks induces them to reduce their intermediating activities which makes the reallocation of capital vulnerable to shifts in Z .

⁴⁰Observe that in the baseline model in which $\mathbb{P}(A_I = 1) = 1$ does not hold there is an additional source of idiosyncratic uncertainty - shocks that affect the productivity level A_I .

Appendix B

Lemma 1

Suppose that i and k_S solve 2. If $A_I > A_I^*(q_S)$ then $i > 0$ and $k_S = k$. If $A_I \leq A_I^*(q_S)$ then $i = 0$ and $0 < k_S < k$.

Proof. Suppose that $A_I > A_I^*(q_S) = \frac{1-\delta}{q_S}$. By contradiction assume that optimal solution to 2 involves: $i \geq 0$ and $0 < k_S < k$. Consider the following deviation from the optimal plan: i-producers sells an additional portion of its capital κ ($0 < \kappa < k - k_S$) and spends a proportion $x = \frac{1-\delta}{A_I q_S}$ of the proceedings κq_S from this transaction on additional investment. Proportion $1 - x > 0$ (it is positive as $A_I > \frac{1-\delta}{q_S}$) is used for increasing consumption. The budget constraint is not violated. Observe that k' does not change:

$$\begin{aligned}\Delta k' &= A_I(i + x\kappa q_S) + (1 - \delta)(k - k_S - \kappa) - A_I i - (1 - \delta)(k - k_S) \\ &= A_I x \kappa q_S - (1 - \delta)\kappa = A_I \frac{1 - \delta}{A_I q_S} \kappa q_S - (1 - \delta)\kappa = 0.\end{aligned}$$

At the same time c increased so this means that plan that involved $i \geq 0$ and $0 < k_S < k$ was not optimal.

Let us consider the case in which $A_I < A_I^*(q_S) = \frac{1-\delta}{q_S}$. Again, by contradiction suppose that optimal solution to 2 involves: $i > 0$ and $0 < k_S \leq k$. Consider the following deviation from the optimal plan: i-producer decreases investment by $0 < \iota < i$ and to guarantee that it budget constraint holds it decreases the amount of capital k that is sold (i.e., k_S) by $\frac{A q_S}{1-\delta} \frac{\iota}{q_S}$. At the same time he consumes the amount $\frac{1-\delta-A q_S}{1-\delta} \iota$ of non-invested goods. As before, k' remains unaffected by this deviation:

$$\begin{aligned}\Delta k' &= A_I(i - \iota) + (1 - \delta)(k + \frac{A q_S}{1 - \delta} \frac{\iota}{q_S} - k_S) - A_I i - (1 - \delta)(k - k_S) \\ &= -A_I \iota + (1 - \delta) \frac{A q_S}{1 - \delta} \frac{\iota}{q_S} = 0.\end{aligned}$$

At the same time, consumption increased so plan that involved $i > 0$ and $0 < k_S \leq k$ is not optimal.

Observe that i-producer remains indifferent between actions that either increase/decrease i and decrease/increase k_S when $A_I = A_I^*(q_S) = \frac{1-\delta}{q_S}$ so that WLOG we set $i = 0$ and $k_S = k$ in such situation. \square

Proposition 1

Decision rules and value function of an i-producer that has productivity level $A_I < A_I^*(q_S)$ are: $c = \frac{\phi}{1+\phi} \omega_{I_0}$, $k' = \frac{1}{1+\phi} \frac{(1-\delta)\omega_{I_0}}{q_S}$, $V^{I_0} = \Psi^{I_0}(K, E) + \left(1 + \frac{1}{\phi}\right) \log \omega_{I_0}$ where $\omega_{I_0} = q_S k$ and $\phi = \frac{1-\beta}{\beta(\Pi(A_L) + \Pi(A_H)\mathbb{E}Z)}$. Decision rules and value function of an i-producer that has productivity level $A_I \geq A_I^*(q_S)$ are: $c = \frac{\phi}{1+\phi} \omega_{I_P}$, $k' = \frac{1}{1+\phi} A_I \omega_{I_P}$, $V^{I_P} = \Psi^{I_P}(K, E, A_I) + \left(1 + \frac{1}{\phi}\right) \log \omega_{I_P}$ where

$\omega_{I_P} = q_S k$. Decision rules and value function of a c-producer are: $c = \frac{\phi Z}{1+\phi Z} \omega_C$, $k' = \frac{1}{1+\phi Z} \frac{(1-\delta)\omega_C}{q_B}$, $V^C = \Psi^C(K, E, Z) + \left(Z + \frac{1}{\phi}\right) \log \omega_C$ where $\omega_C = (G(K) + q_B) k$.

Proof. Let us prove the case of the i-producer that has productivity level $A_I \geq A_I^*(q_S)$. The remaining cases are analogous and I will omit them.

First, calculate i from the law of motion and plug it into the budget constraint. I get:

$$c + \frac{k'}{A_I} = q_S k.$$

Let us denote $\omega_{I_P} = q_S k$. This transforms our problem into a standard consumption-savings problem and enables me to use arguments presented by Alvarez and Stokey [Alvarez and Stokey (1998)] regarding dynamic programming problem with homogeneous objective function (in particular, solution to Bellman equation is unique).

To prove the exact functional forms of policies listed in Proposition 1, I proceed by guess and verify method. Let us substitute the guesses of V^{I_P} , V^{I_0} , V^C into i-producer's (that has $A_I \geq A_I^*(q_S)$) Bellman equation:

$$\begin{aligned} V^{I_P}(k, K, E, A_I) = \max_{c > 0, i \geq 0, k' > 0} & \left\{ \log(c) + \beta \mathbb{E}_{Z, Z', A'_I} \left(\pi_I \cdot \mathbb{P}_{A_I} (A_I \geq A_I^*(q'_S)) \cdot \left(\Psi^{I_P}(K', E', A'_I) + \left(1 + \frac{1}{\phi}\right) \log \omega'_{I_P} \right) \right. \right. \\ & + \pi_I \cdot \mathbb{P}_{A_I} (A_I < A_I^*(q'_S)) \cdot \left(\Psi^{I_0}(K', E') + \left(1 + \frac{1}{\phi}\right) \log \omega'_{I_0} \right) \\ & \left. \left. + \pi_C \cdot \left(\Psi^C(K', E', Z') + \left(Z' + \frac{1}{\phi}\right) \log \omega'_C \right) | K, E \right) \right\}. \end{aligned}$$

subject to :

$$\begin{cases} c + \frac{k'}{A_I} = q_S k, \\ E' = E'(K, E, Z), \\ K' = K'(K, E), \end{cases}$$

By the fact that $\log \omega'_{I_P} = \log q'_S + \log k'$ (similarly for $\log \omega'_{I_0}$ and $\log \omega'_C$) I get:

$$V^{I_P}(k, K, E, A_I) = \max_{c > 0, k'} \log(c) + \frac{1}{\phi} \log k' + \bar{\Psi}^{I_P}(K, E)$$

subject to :

$$c + \frac{k'}{A_I} = \omega_{I_P}.$$

FOC is:

$$k' = \frac{1}{1 + \phi} \cdot A_I \omega_{I_P}.$$

From the budget constraint we get:

$$c = \frac{\phi}{1 + \phi} \omega_{I_P},$$

which confirms our guess for decision rules. I plug solutions for c and k' back to Bellman equation:

$$V^{I_P} = \Psi^{I_P}(K, E, A_I) + \left(1 + \frac{1}{\phi}\right) \log \omega_{I_P}$$

which completes the proof. \square

Proposition 2

If A1 holds then decision rules and value function of intermediary are: $c = (1 - \beta)\omega_F$, $e' = \beta\omega_F$, $W_2 = \Psi^F(K, E, Z) + \frac{1}{1-\beta} \log \omega_F$, $k_F = \Phi(K, E) \cdot e$, where $\omega_F = e + (q_B - q_S)k_F$.

Proof. The method used to prove Proposition 2 is analogous to one that was used to show that Proposition 1 holds. There is however one additional issue that needs to be solved: we need to show that the budget constraint

$$c + e' = e + (q_B - q_S)k_F$$

can be rearranged to the form of a constraint that is present in the standard consumption-savings problem. To prove that, let us first plug the guess for W_2 into W_1 :

$$W_1(e, K, E) = \max_{k_F} \mathbb{E}_Z \left(\Psi^F(K, E, Z) + \frac{1}{1-\beta} \log \omega_F \right),$$

substituting $\omega_F = e + (q_B(K, E, Z) - q_S(K, E))k_F$ I get:

$$W_1(e, K, E) = \max_{k_F} \mathbb{E}_Z \left(\Psi^F(K, E, Z) + \frac{1}{1-\beta} \log (e + (q_B(K, E, Z) - q_S(K, E))k_F) \right).$$

The FOC under A1 reads:

$$\begin{aligned} & \pi(Z_L) \cdot \frac{q_B(K, E, Z_L) - q_S(K, E)}{e + [q_B(K, E, Z_L) - q_S(K, E)] k_F} \\ & + \pi(Z_H) \cdot \frac{q_B(K, E, Z_H) - q_S(K, E)}{e + [q_B(K, E, Z_H) - q_S(K, E)] k_F} = 0. \end{aligned} \tag{33}$$

After a reformulation we get:

$$\begin{aligned} k_F &= \left(\frac{\pi(Z_H)}{q_S(K, E) - q_B(K, E, Z_L)} - \frac{\pi(Z_L)}{q_B(K, E, Z_H) - q_S(K, E)} \right) e \\ &= \mathbb{E}_Z \left(\frac{\pi(Z)}{q_S(K, E) - q_B(K, E, Z)} \right) e \end{aligned}$$

which verifies my guess: $k_F = \Phi(K, E) \cdot e$. Let us show that k_F is positive. Observe that it is true iff:

$$\frac{\pi(Z_H)}{q_S(K, E) - q_B(K, E, Z_L)} > \frac{\pi(Z_L)}{q_B(K, E, Z_H) - q_S(K, E)},$$

which is equivalent to:

$$\pi(Z_H) (q_B(K, E, Z_H) - q_S(K, E)) + \pi(Z_L) (q_B(K, E, Z_L) - q_S(K, E)) > 0. \quad (34)$$

I will show later, that in equilibrium: $q_B(K, E, Z_H) - q_S(K, E) < 0$ and $q_B(K, E, Z_L) - q_S(K, E) > 0$. Additionally, I can write the FOC 33 in the following form:

$$\mathcal{C}_1 \pi(Z_H) (q_B(K, E, Z_H) - q_S(K, E))$$

$$+ \mathcal{C}_2 \pi(Z_L) (q_B(K, E, Z_L) - q_S(K, E)) = 0,$$

where $\mathcal{C}_1 > \mathcal{C}_2$ (because $q_B(K, E, Z_H) - q_S(K, E) < 0$ and $q_B(K, E, Z_L) - q_S(K, E) > 0$ in equilibrium). This implies that:

$$\pi(Z_H) (q_B(K, E, Z_H) - q_S(K, E))$$

$$+ \frac{\mathcal{C}_2}{\mathcal{C}_1} \pi(Z_L) (q_B(K, E, Z_L) - q_S(K, E)) = 0,$$

where $\frac{\mathcal{C}_2}{\mathcal{C}_1} < 1$. But this means that 34 holds as the "weight" of 1 given to a positive term $q_B(K, E, Z_L) - q_S(K, E) > 0$ is higher than $\frac{\mathcal{C}_2}{\mathcal{C}_1}$ in the equation above.

We are now in position to finish the proof in a standard way which was used for verification of policies and value functions of entrepreneurs. First note that since $k_F = \Phi(K, E) \cdot e$ then:

$$W_2(k_F, e, K, E, Z) = \tilde{W}_2(e, K, E, Z).$$

This means that:

$$\tilde{W}_2(e, K, E, Z) = \max_{c, e'} (\log c + \beta W_1(e', K', E'))$$

subject to :

$$c + e' = (1 + (q_B - q_S)\Phi(K, E))e = \omega_F,$$

$$E' = e'.$$

$$K' = K'(K, E).$$

I plug my guess for W_2 into W_1 and to the equation above:

$$\tilde{W}_2(e, K, E, Z) = \max_{c, e'} \left(\log c + \beta \mathbb{E}_{Z'} \left(\Psi^F(K', E', Z') + \frac{1}{1 - \beta} \log(e' + (q_B - q_S)\Phi(K', E')e') \right) \right)$$

subject to

$$c + e' = \omega_B.$$

$$E' = e'$$

$$K' = K'(K, E).$$

This means that:

$$\tilde{W}_2(e, K, E, Z) = \max_{c, e'} \left(\log c + \frac{\beta}{1 - \beta} \log e' + \tilde{\Psi}^F(K, E, Z) \right),$$

$$c + e' = \omega_F.$$

First order conditions are: $e' = \beta \omega_F$ and $c = (1 - \beta) \omega_F$. This confirms my guess for policy functions. We plug them back into Bellman equations to get:

$$\tilde{W}_2(e, K, E, Z) = \frac{1}{1 - \beta} \log \omega_F + \Psi^F(K, E, Z),$$

but we know that $\omega_F = e + (q_B - q_S)k_F$ (i.e., ω_F is a function of k_F) so I can return to the initial formulation of W_2 :

$$W_2(k_F, e, K, E, Z) = \Psi^F(K, E, Z) + \frac{1}{1 - \beta} \log \omega_F$$

and this completes the proof. □

Lemma 2

The following inequalities hold in equilibrium:

$$D^{-1}(S(q_S), K, Z_H) < q_S,$$

$$D^{-1}(S(q_S), K, Z_L) > q_S.$$

Proof. I will prove Lemma 2 by contradiction: Suppose that in equilibrium:

$$D^{-1}(S(q_S), K, Z_H) \geq q_S$$

$$D^{-1}(S(q_S), K, Z_L) > q_S.$$

This implies that (by the market clearing conditions in Definition 1): $q_B(Z_H) \geq q_S$ and $q_B(Z_L) > q_S$ (I omit arguments K, E of q_B for clarity of exposition) but then banks have incentives to increase k_F which cannot happen in equilibrium.

Suppose that in equilibrium:

$$D^{-1}(S(q_S), K, Z_H) < q_S$$

$$D^{-1}(S(q_S), K, Z_L) \leq q_S.$$

This implies: $q_B(Z_H) < q_S$ and $q_B(Z_L) \leq q_S$ but then banks have incentives to decrease k_F which cannot happen in equilibrium.

Suppose that in equilibrium:

$$D^{-1}(S(q_S), K, Z_H) > q_S$$

$$D^{-1}(S(q_S), K, Z_L) < q_S.$$

This implies that $D^{-1}(S(q_S), K, Z_L) < D^{-1}(S(q_S), K, Z_H)$ and contradicts the fact that D is strictly decreasing in Z . Same argument excludes the possibility that:

$$D^{-1}(S(q_S), K, Z_H) = q_S$$

$$D^{-1}(S(q_S), K, Z_L) = q_S.$$

This completes the proof. □

Theorem 1

If A1 holds then solution to equation 13 exists and is unique.

Proof. Let us prove existence first. I reformulate the equilibrium condition 13 to get:

$$\begin{aligned} & \pi_H \frac{e + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S)}{e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)} \\ &= -(1 - \pi_H) \frac{D^{-1}(S(q_S), K, Z_L) - q_S}{D^{-1}(S(q_S), K, Z_H) - q_S}, \end{aligned} \quad (35)$$

where $\pi_L = \pi(Z_L)$ and $\pi_H = \pi(Z_H)$. I omit argument of S (i.e., argument K) to economize on notation. This reformulation was possible since by Lemma 2 $D^{-1}(S(q_S), K, Z_H) - q_S \neq 0$ and by the log specification of preferences the non-zero consumption in problem 10 implies that $e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S) \neq 0$.

By $\underline{q_S}$ denote q_S that satisfies:

$$D^{-1}(S(q_S), K, Z_H) - q_S = 0.$$

This number exists because there exists value q_S (because $D(q_S, K, Z_H)$ and $S(q_S)$ intersect only once - see Figure 5) such that: $S(q_S) = D(q_S, K, Z_H)$ (and this implies the existence of q_S that solves $D^{-1}(S(q_S), K, Z_H) = q_S$). Notice that for q_S converging to $\underline{q_S}$ from above, the LHS of the reformulated

Figure 9: Theorem 1 - existence, case $\bar{q}_{S,1} \leq \bar{q}_{S,2}$

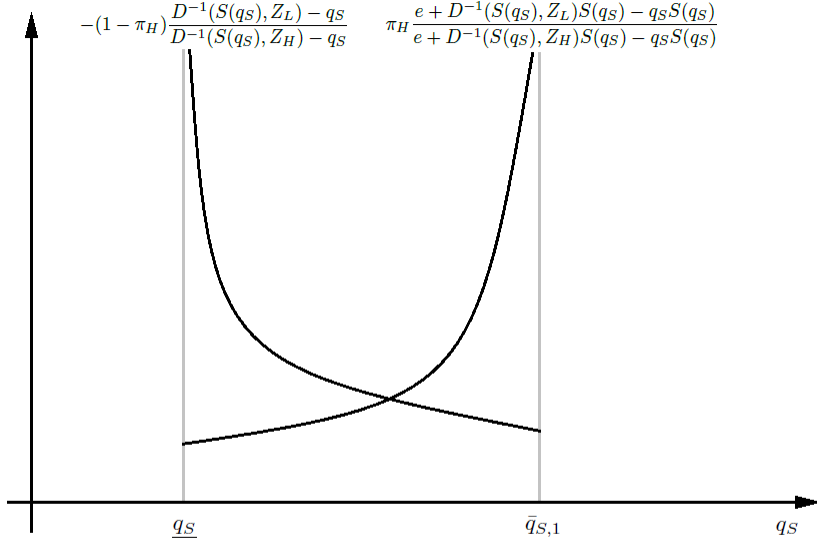
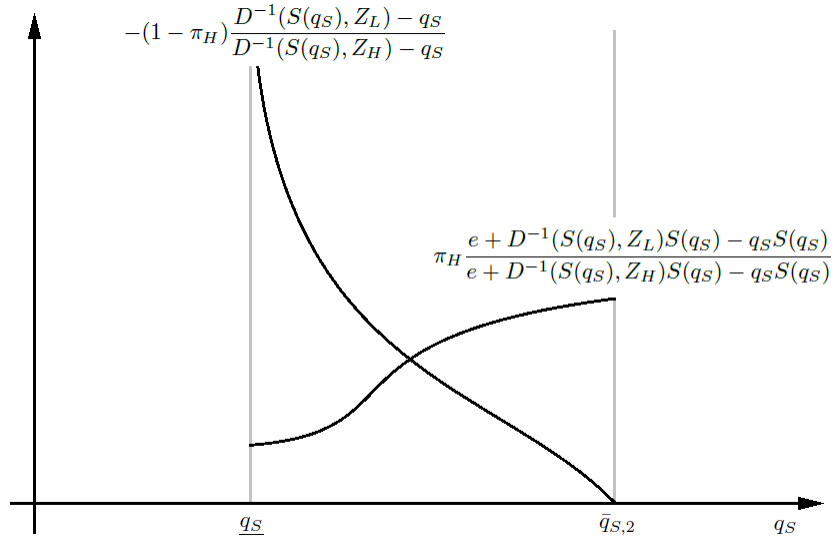


Figure 10: Theorem 1 - existence, case $\bar{q}_{S,1} > \bar{q}_{S,2}$



equilibrium condition 35 is a finite positive number and the RHS converges to $+\infty$ (as the denominator is negative by Lemma 2).

Now let us define two additional numbers that are strictly greater than \underline{q}_S : the first one, $\bar{q}_{S,1}$ solves:

$$e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S) = 0. \quad (36)$$

There exists such a number greater than \underline{q}_S because the LHS of 36 evaluated at \underline{q}_S is equal to $e > 0$. On the other hand since $\lim_{q_S \rightarrow +\infty} S(q_S) = \pi_I \cdot K$ and hence $\lim_{q_S \rightarrow +\infty} D^{-1}(S(q_S), K, Z_H)$ is a finite positive number, then the LHS of 36 converges to $-\infty$ as $q_S \rightarrow +\infty$. This means that $\bar{q}_{S,1}$ exists by the Mean Value Property (since the LHS of 36 is continuous). Observe that if q_S converges to $\bar{q}_{S,1}$ then the LHS approaches to $+\infty$ and the RHS is a finite positive number. The second one is: $\bar{q}_{S,2}$ that solves:

$$D^{-1}(S(q_S), K, Z_L) - q_S = 0,$$

existence of which is guaranteed by identical reasons as those presented for \underline{q}_S (observe that if the intersection of $D(q_S, K, Z_H)$ and $S(q_S)$ is well defined then the intersection of $D(q_S, K, Z_L)$ and $S(q_S)$ exists, too).

Let us consider two cases: $\bar{q}_{S,1} > \bar{q}_{S,2}$ and $\bar{q}_{S,2} \geq \bar{q}_{S,1}$. If $\bar{q}_{S,2} \geq \bar{q}_{S,1}$ then from what was said above the two continuous curves defined by the RHS and the LHS of the reformulated FOC 35 must intersect at some point $q_S^* \in (\underline{q}_S, \bar{q}_{S,1})$ as one of them converges to $+\infty$ at one end of this interval while the other is positive (not necessarily strictly positive) and the situation is the other way round on the other end of the interval. If $\bar{q}_{S,1} > \bar{q}_{S,2}$ it can be observed that for q_S converging to \underline{q}_S the RHS goes to $+\infty$ and the LHS is strictly positive. For q_S converging to $\bar{q}_{S,2}$ the RHS converges to 0 while the LHS approaches to a strictly positive number. Since they are both continuous for $(\underline{q}_S, \bar{q}_{S,2})$ then they must intersect at some point $q_S^* \in (\underline{q}_S, \bar{q}_{S,2})$. This means that a solution to 13 exists.

Let us prove uniqueness now. I will be using another form of 13:

$$(1 - \pi_H) \frac{D^{-1}(S(q_S), K, Z_L) - q_S}{e + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S)} = \pi_H \frac{q_S - D^{-1}(S(q_S), K, Z_H)}{e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)}. \quad (37)$$

Let us analyze the RHS of the reformulated FOC 37 now. It can be calculated that:

$$\begin{aligned} & \left(\frac{D^{-1}(S(q_S), K, Z_H) - q_S}{e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)} \right)' \\ &= \frac{1}{(e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S))^2} \\ & \cdot \left\{ \left(\frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S), K, Z_H), Z_H)} - 1 \right) \cdot (e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)) \right. \\ & \left. - \left[S'(q_S) \cdot (D^{-1}(S(q_S), K, Z_H) - q_S) + S(q_S) \cdot \left(\frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S), K, Z_H), Z_H)} - 1 \right) \right] \cdot (D^{-1}(S(q_S), K, Z_H) - q_S) \right\} \end{aligned}$$

$$= \frac{1}{(e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S))^2} \cdot \left\{ \left(\frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S), K, Z_H), Z_H)} - 1 \right) \cdot e - S'(q_S) (D^{-1}(S(q_S), K, Z_H) - q_S)^2 \right\} < 0 \quad (38)$$

This is because $e > 0$, $S'(q_S) > 0$ and $D_{q_B} < 0$. This implies that the RHS is an increasing function of q_S . It is easy to see that analogous calculations prove that the LHS is a decreasing function of q_S .

This means that the RHS and the LHS of 37 intersect at most once. But by our previous considerations we know that they do intersect so the point of the intersection is unique. \square

Claim 1

Aggregate reallocation of capital k_F increases with E .

Proof. From the bank's FOC and $E = e$ we get:

$$(1 - \pi_H) \frac{D^{-1}(S(q_S), K, Z_L) - q_S}{E + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S)} + \pi_H \frac{D^{-1}(S(q_S), K, Z_H) - q_S}{E + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)} = 0. \quad (39)$$

Let us denote the LHS of 39 by $B(q_S, E)$ (I can ignore the second state variable - K as it is chosen in the first stage of the previous period and hence it remains unaffected by the choice of E in the second stage of the previous period) in From the proof of uniqueness we know that $B_{q_S}(q_S, E) < 0$. Let us check the sign of $B_E(q_S, E)$ now. I calculate:

$$B_E(q_S, E) = -(1 - \pi_H) \frac{D^{-1}(S(q_S), K, Z_L) - q_S}{(E + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S))^2} - \pi_H \frac{D^{-1}(S(q_S), K, Z_H) - q_S}{(E + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S))^2}. \quad (40)$$

Since $B_E(q_S, E)$ is evaluated in equilibrium then bank's FOC must hold and then I can substitute $\pi_H \frac{D^{-1}(S(q_S), K, Z_H) - q_S}{E + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)}$ for $(1 - \pi_H) \frac{D^{-1}(S(q_S), K, Z_L) - q_S}{E + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S)}$ in 40 to get:

$$B_E(q_S, E) = \pi_H \frac{D^{-1}(S(q_S), K, Z_H) - q_S}{E + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)} \cdot \left(\frac{1}{E + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S)} - \frac{1}{E + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)} \right).$$

Observe that since by Lemma 2 $D^{-1}(S(q_S), K, Z_H) - q_S < 0$ and by the fact that:

$$E + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S) > E + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)$$

value $B_E(q_S, E)$ evaluated in equilibrium is positive. I use the Implicit Function Theorem to obtain:

$$k'_F(E) > 0.$$

This completes the proof. □

Lemma 3

Condition 24 holds for all parameter values.

Proof. Let us rewrite the condition that we want to prove:

$$\forall_K \frac{G(K)}{(1 + \phi Z_H)^{\frac{\pi_I}{\pi_C}} + \phi Z_H} > 1 - \delta. \quad (41)$$

My strategy is the following: I find the upper bound for K (I denote it by \tilde{K}) in the dynamic model. Then I prove that 41 holds for \tilde{K} . Then I use the fact that G decreases in K and hence I get the result for all K .

First, let us find \tilde{K} . Observe that the rate of aggregate investment satisfies:

$$I(q_S, K) = \frac{q_S \pi_I K}{1 + \phi} < \frac{q_B(Z_L) \pi_I K}{1 + \phi} = I(q_B(Z_L), K).$$

It is because in equilibrium $q_S < q_B(Z_L)$. It is clear (from 28 and from 6) that $q_B(Z_L)$ depends solely on one state variable, i.e. K so we do not need to keep track of E in the further considerations. Suppose that the economy experiences an infinitely long path of "good shocks" $Z = Z_L$. This means that (if we assume that K_0 is sufficiently small) under investment $I(q_B(Z_L), K)$ the aggregate capital K converges to steady state characterized by the following equation:

$$I(q_B(Z_L), K) = \delta K. \quad (42)$$

This steady state is our candidate \tilde{K} . We calculate (I use the inverse demand function to replace $q_B(Z_L)$):

$$\begin{aligned} I(q_B(Z_L), K) &= \frac{q_B(Z_L) \pi_I K}{1 + \phi} \\ &= \frac{G(K)}{(1 + \phi Z_H)^{\frac{\pi_I}{\pi_C}} + \phi Z_H} \cdot \frac{\pi_I K}{1 + \phi} \end{aligned}$$

$$= \frac{(1-\alpha)^{\frac{1}{\alpha}} \left(\frac{\alpha}{1-\alpha} \right) A_C^{\frac{1}{\alpha}} (1-\alpha)^{\frac{\alpha-1}{\alpha}} A_C^{\frac{\alpha-1}{\alpha}} (\pi_C K)^{\alpha-1} L^{1-\alpha}}{(1+\phi Z_H)^{\frac{\pi_I}{\pi_C}} + \phi Z_H} \cdot \frac{\pi_I K}{1+\phi}.$$

We use 42 to compute \tilde{K} :

$$\tilde{K} = \left[\frac{\alpha A_C \pi_I}{\delta \left((1+\phi Z_L)^{\frac{\pi_I}{\pi_C}} + \phi Z_L \right) (1+\phi)} \right]^{\frac{1}{1-\alpha}} \frac{L}{\pi_C}.$$

Now I show that 41 holds for \tilde{K} .

$$\begin{aligned} \frac{G(\tilde{K})}{(1+\phi Z_H)^{\frac{\pi_I}{\pi_C}} + \phi Z_H} &> 1 - \delta \\ \iff \left[\frac{(1+\phi Z_L)^{\frac{\pi_I}{\pi_C}} + \phi Z_L}{(1+\phi Z_H)^{\frac{\pi_I}{\pi_C}} + \phi Z_H} \right] \frac{1+\phi}{\pi_I} &> \frac{1}{\delta} - 1 \end{aligned}$$

which is implied by our assumption about parameter values 23. Since \tilde{K} is an upper bound for all capital values then by the fact that G decreases with K we have:

$$\forall_K \frac{G(K)}{(1+\phi Z_H)^{\frac{\pi_I}{\pi_C}} + \phi Z_H} > 1 - \delta$$

which completes the proof. \square

Theorem 1.8

If 23 holds then solution to equation 29 exists and is unique.

Proof. Let us rewrite the equilibrium condition 29:

$$\begin{aligned} \pi(Z_L) \cdot \frac{D^{-1}(K, Z_L) - q_S}{E + [D^{-1}(K, Z_L) - q_S] \pi_I K} \\ \pi(Z_H) \cdot \frac{D^{-1}(K, Z_H) - q_S}{E + [D^{-1}(K, Z_H) - q_S] \pi_I K} = 0. \end{aligned} \tag{43}$$

It is clear that we need to consider values of q_S that satisfy: $q_S \in (D^{-1}(K, Z_H), D^{-1}(K, Z_L))$ (by a similar reasoning to the one captured by Lemma 2). The LHS of 43 is continuous for $q_S \in (D^{-1}(K, Z_H), \min\{D^{-1}(K, Z_L), \bar{q}_S\})$ where \bar{q}_S solves:

$$\begin{aligned} E + [D^{-1}(K, Z_H) - \bar{q}_S] \pi_I K &= 0 \\ \implies \bar{q}_S &= D^{-1}(K, Z_H) + \frac{E}{\pi_I K}. \end{aligned}$$

For $q_S = D^{-1}(K, Z_H)$ the LHS of 43 is positive. Suppose that $\min\{D^{-1}(K, Z_L), \bar{q}_S\} = D^{-1}(K, Z_L)$ then the LHS of 43 is negative. If $\min\{D^{-1}(K, Z_L), \bar{q}_S\} = \bar{q}_S$ then the LHS of 43 converges to $-\infty$ for $q_S \rightarrow \bar{q}_S$. This means that by the Mean Value Theorem, solution to 43 exists.

Let us prove uniqueness now. Let us concentrate on the derivative of $\frac{D^{-1}(K, Z) - q_S}{E + [D^{-1}(K, Z) - q_S] \pi_I K}$ now:

$$\begin{aligned} & \left(\frac{D^{-1}(K, Z) - q_S}{E + [D^{-1}(K, Z) - q_S] \pi_I K} \right)' \\ &= \frac{-E}{(E + [D^{-1}(K, Z) - q_S] \pi_I K)^2} < 0. \end{aligned}$$

This means that the LHS of 43 is strictly decreasing. This and existence of q_S that satisfies 43 means that this solution is unique. \square

Claim 1.8

Price q_S paid by banks for capital bought from i-producers increases in E (for K kept constant).

Proof. We will apply the Implicit Function Theorem to 29. From the proof of Theorem 1.8 we know that the derivative of the LHS of 29 decreases with q_S . Derivative of the LHS of 29 with respect to E is:

$$-\left\{ \pi_H \frac{D^{-1}(K, Z_H) - q_S}{E + [D^{-1}(K, Z_H) - q_S] \pi_I K} \cdot \left(\frac{1}{E + [D^{-1}(K, Z_H) - q_S] \pi_I K} - \frac{1}{E + [D^{-1}(K, Z_L) - q_S] \pi_I K} \right) \right\} < 0.$$

This implies that $(q_S(K, E))'_E > 0$. \square

Proposition 3

If A1 holds then decision rules and value function of monopolistic intermediary are: $c = (1 - \beta)\omega_F$, $e' = \beta\omega_F$, $W_2 = \Psi^F(K, E, Z) + \frac{1}{1-\beta} \log \omega_F$, $k_{F,j} = \tilde{\Phi}(K, E) \cdot e$, where $\omega_F = e + (q_B - q_S) k_{F,j}$.

Proof. It is sufficient to show that the FOC with respect to $k_{F,j}$ of the following expression:

$$W_1(e, K, E) = \max_{k_{F,j}} \mathbb{E}_Z \left(\Psi^F(K, E, Z) + \frac{1}{1-\beta} \log \left(e + \left(q_B \cdot \left(\frac{k_F}{k_{F,j}} \right)^{1-\frac{1}{\epsilon}} - q_S \right) k_{F,j} \right) \right),$$

defines an implicit, linear relationship between e and $k_{F,j}$ - the rest of the proof is done exactly in the same way as in proof of Proposition 2.

The FOC reads:

$$\pi(Z_L) \cdot \frac{\frac{1}{\epsilon} q_B(K, E, Z_L) \left(\frac{k_{F,j}}{k_F} \right)^{\frac{1}{\epsilon}-1} - q_S(K, E)}{e + \left[q_B(K, E, Z_L) \left(\frac{k_{F,j}}{k_F} \right)^{\frac{1}{\epsilon}-1} - q_S(K, E) \right] k_F}$$

$$\pi(Z_H) \cdot \frac{\frac{1}{\epsilon} q_B(K, E, Z_H) \left(\frac{k_{F,j}}{k_F} \right)^{\frac{1}{\epsilon}-1} - q_S(K, E)}{e + \left[q_B(K, E, Z_H) \left(\frac{k_{F,j}}{k_F} \right)^{\frac{1}{\epsilon}-1} - q_S(K, E) \right] k_F} = 0. \quad (44)$$

Since I consider the symmetric case in which $e = E$ and rational agents recognize that their decisions are identical then they know that $k_{F,j} = k_F$ and hence the FOC is:

$$\begin{aligned} \pi(Z_L) \cdot \frac{\frac{1}{\epsilon} q_B(K, E, Z_L) - q_S(K, E)}{e + [q_B(K, E, Z_L) - q_S(K, E)] k_F} \\ \pi(Z_H) \cdot \frac{\frac{1}{\epsilon} q_B(K, E, Z_H) - q_S(K, E)}{e + [q_B(K, E, Z_H) - q_S(K, E)] k_F} = 0. \end{aligned} \quad (45)$$

Observe that 45 implies that there exists a linear relationship between $k_{F,j}$ and e : $k_{F,j} = \tilde{\Phi}(K, E) \cdot e$. This in turn means that the budget constraint can be reformulated:

$$\omega_F = e + (q_B - q_S) \tilde{\Phi}(K, E) \cdot e$$

and hence the problem of the monopolistic intermediary becomes a standard consumption-savings problem. \square

Theorem 2

Under A1 solution to equation 20 exists and is unique.

Proof. First, observe that analogously to Lemma 2, marginal profit from intermediation in state Z_H : $MP(Z_H) = \pi(Z_H) \cdot \frac{\frac{1}{\epsilon} q_B(K, E, Z_H) - q_S(K, E)}{e + [q_B(K, E, Z_H) - q_S(K, E)] k_F}$ is negative and marginal profit from intermediation in state Z_L : $MP(Z_L) = \pi(Z_L) \cdot \frac{\frac{1}{\epsilon} q_B(K, E, Z_L) - q_S(K, E)}{e + [q_B(K, E, Z_L) - q_S(K, E)] k_F}$ is positive. If, by contradiction, $MP(Z_L) < 0 < MP(Z_H)$ then it violates the relationship:

$$q_B(K, E, Z_H) = D^{-1}(S(q_S(K, E)), K, Z_H) < D^{-1}(S(q_S(K, E)), K, Z_L) = q_B(K, E, Z_L) \quad (46)$$

which is implied by $D_Z < 0$. If, by contradiction $MP(Z_L) > 0$ and $MP(Z_H) \geq 0$ or $MP(Z_L) < 0$ and $MP(Z_H) \leq 0$ then equality described by bank's FOC is violated. It is violated also for $MP(Z_L) = 0$ and $MP(Z_H) < 0$ and by $MP(Z_H) = 0$ and $MP(Z_L) > 0$. Observe that 46 excludes the possibility that $MP(Z_H) = 0$ and $MP(Z_L) = 0$. This implies that if equilibrium exists then the following relationship must hold:

$$MP(Z_L) > 0 > MP(Z_H). \quad (47)$$

Since logarithmic preferences imply: $e + [q_B(K, E, Z_H) - q_S(K, E)] k_F > 0$ and $e + [q_B(K, E, Z_L) - q_S(K, E)] k_F > 0$ then 47 implies:

$$\frac{1}{\epsilon} q_B(K, E, Z_L) - q_S(K, E) > 0 > \frac{1}{\epsilon} q_B(K, E, Z_H) - q_S(K, E).$$

We are in position to prove existence of equilibrium. It can be done in an analogous way as in proof of existence of solution to 13, with the only difference that $\underline{q_S}$ is defined as q_s that satisfies:

$$S(q_S, K) = D(\epsilon q_S, K, Z_H)$$

and $\bar{q}_{S,2}$ is q_s that solves:

$$S(q_S, K) = D(\epsilon q_S, K, Z_L).$$

This means that solution to 20 exists.

Let us consider uniqueness now. Reformulated equilibrium condition 20 is:

$$\begin{aligned} (1 - \pi_H) \frac{\frac{1}{\epsilon} D^{-1}(S(q_S), K, Z_L) - q_S}{e + D^{-1}(S(q_S), K, Z_L) S(q_S) - q_S S(q_S)} = \\ \pi_H \frac{q_S - \frac{1}{\epsilon} D^{-1}(S(q_S), K, Z_H)}{e + D^{-1}(S(q_S), K, Z_H) S(q_S) - q_S S(q_S)}. \end{aligned} \quad (48)$$

Let us calculate:

$$\begin{aligned} & \left(\frac{\frac{1}{\epsilon} D^{-1}(S(q_S), K, Z_H) - q_S}{e + D^{-1}(S(q_S), K, Z_H) S(q_S) - q_S S(q_S)} \right)' \\ &= \frac{1}{(e + D^{-1}(S(q_S), K, Z_H) S(q_S) - q_S S(q_S))^2} \\ & \cdot \left\{ \left(\frac{1}{\epsilon} \frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S), K, Z_H), Z_H)} - 1 \right) \cdot e \right. \\ & \quad - S'(q_S) (D^{-1}(S(q_S), K, Z_H) - q_S)^2 \\ & \quad - (1 - \frac{1}{\epsilon}) \cdot D^{-1}(S(q_S), K, Z_H) \cdot S(q_S) \\ & \quad \left. + \left(1 - \frac{1}{\epsilon} \right) \cdot q_S \cdot S(q_S) \cdot \frac{S'(q_S)}{D_{q_B}(D^{-1}(S(q_S), K, Z_H), Z_H)} \right\} < 0. \end{aligned}$$

It is because all terms in braces are negative (by the fact that $S' > 0$, $D_{q_B} < 0$, $D^{-1} > 0$, $S > 0$ and $\epsilon > 1$). This means that the LHS of 48 decreases in q_S and the RHS increases in q_S . Since we know that they intersect (by existence) it means that solution to 48 is unique. \square

Claim 2

Aggregate reallocation of capital k_F increases with E in RCE with monopolistically competitive intermediaries.

Proof. Proof is almost identical to the case of RCE with perfectly competitive banks. Steps are the same, the Implicit Function Theorem is used. I only show that the partial derivative of the LHS of 20 with respect to e reads:

$$\pi_H \frac{\frac{1}{\epsilon} D(S(q_S), K, Z_H) - q_S}{E + D^{-1}(S(q_S), K, Z_H) S(q_S) - q_S S(q_S)}$$

$$\cdot \left(\frac{1}{E + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S)} - \frac{1}{E + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)} \right) > 0.$$

Since (by the proof of uniqueness) the partial derivative of the LHS of 20 with respect to q_S is negative. Hence by the Implicit Function Theorem $k'_F(E) > 0$. \square

Proposition 4

If the initial value of aggregate intermediaries' equity E and aggregate capital K are the same in both economies: the one with competitive banks and the one with monopolistically competitive intermediaries, then the amount of intermediated capital is strictly higher in economy with competitive banks than in economy with monopolistically competitive intermediaries.

Proof. It suffices to investigate equilibrium conditions 13 and 20. Let us reformulate them to get:

$$\begin{aligned} (1 - \pi_H) \frac{\frac{1}{\epsilon} D^{-1}(S(q_S), K, Z_L) - q_S}{e + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S)} = \\ \pi_H \frac{q_S - \frac{1}{\epsilon} D^{-1}(S(q_S), K, Z_H)}{e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)}, \end{aligned} \quad (49)$$

for economy with monopolistically competitive banks and:

$$\begin{aligned} (1 - \pi_H) \frac{D^{-1}(S(q_S), K, Z_L) - q_S}{e + D^{-1}(S(q_S), K, Z_L)S(q_S) - q_S S(q_S)} = \\ \pi_H \frac{q_S - D^{-1}(S(q_S), K, Z_H)}{e + D^{-1}(S(q_S), K, Z_H)S(q_S) - q_S S(q_S)}, \end{aligned} \quad (50)$$

for economy with competitive intermediaries. From proofs of Theorems 1 and 2 we know that the LHS of 50 can be treated as decreasing function of q_S . On the other hand the RHS of 50 increases in q_S . Analogous results hold for the RHS and the LHS of 49. It is immediate that the curve defined by the LHS of 49 is strictly below the curve defined by the LHS of 50 since $\frac{1}{\epsilon} < 1$. On the other hand the curve defined by the RHS of 49 is strictly above the one defined by the RHS 50. This implies that the point of intersection described by 49 - q_S^{MC} is smaller than q_S^C that solves 50. But this means that:

$$k_F^{MC} = S(q_S^{MC}) < S(q_S^C) = k_F^C,$$

which completes the proof. \square

Proposition 6

The common lower bound on the supports of ergodic densities associated with E^C and E^{MC} is 0.

Proof. Let us assume that upper bounds on densities' supports of E^C and E^{MC} exist (this will be shown in subsequent propositions). Let's denote them by \bar{E}^C and \bar{E}^{MC} . Take an arbitrarily small

number $\mu > 0$. The idea of the proof (for the lower bounds \bar{E}^C and \bar{E}^{MC}) is to show that with some positive probability there exists a sufficiently long path of adverse shocks $\{Z_H, Z_H, \dots, Z_H\}$ that the corresponding path of E_t^C (or WLOG the path of E_t^{MC}) decreases below μ . Then it is argued (by the Borel-Cantelli lemma) that for almost all trajectories $\{Z_t\}_{t=0}^{+\infty}$ there is an infinite number of such sequences $\{Z_H, Z_H, \dots, Z_H\}$ and since the economy starts (i.e., when such sequence begins) from the lower level of E^C than \bar{E}^C then the corresponding path of E_t^C will decrease below μ as well. Then by the fact that μ is arbitrary and that the number of these paths of $\{Z_H, Z_H, \dots, Z_H\}$ is infinite we can argue that the value of density associated with the ergodic distribution of E^C is strictly positive for all positive numbers in the neighborhood of 0.

Let us consider the economy that starts at \bar{K}^C and \bar{E}^C in period 0. If it is affected by an adverse shock in this period then the next period's value of E is:

$$E_1^C = \beta \cdot (\bar{E}^C + (q_{B,1}(Z_H) - q_S) \cdot k_F) < \beta \bar{E}^C.$$

This inequality follows because for $Z = Z_H$ margin $q_{B,1} - q_S$ is negative in equilibrium. Using the same argument it is easy to see that:

$$E_t^C < \beta^t \bar{E}^C.$$

This means that there exists $t = T$ such that $E_T^C < \beta^T \bar{E}^C < \mu$ (because $\beta \in (0, 1)$). This means that with probability $(\mathbb{P}(Z = Z_H))^T > 0$ economy that starts \bar{K}^C and \bar{E}^C in period 0 has bank's equity lower than μ in period T . Now, by the Borel-Cantelli lemma we know that with probability 1 there is an infinite number of sequences $\{Z_H, Z_H, \dots, Z_H\}$ of length T (within the sequence $\{Z_t\}_{t=0}^{+\infty}$) such that E^C falls below μ (at the end of the corresponding sequence of endogenous state variables) for an infinite number of times. This means that measure of the ergodic distribution of E^C that is accumulated in $(0, \mu)$ is positive. If the ergodic density exists then it means that it is positive for all positive numbers in a small neighborhood of 0. The same reasoning applies for the lower bound of ergodic density associated with E^{MC} . \square

Proposition 7

If $\mathbb{P}(A_I = 1) = 1$ and condition 23 hold then the common lower bound on the supports of ergodic densities associated with K^C and K^{MC} is $\underline{K} = \left(\frac{\Psi}{\delta}\right)^{\frac{1}{1-\alpha}}$ where Ψ is a function of parameters.

Proof. The strategy of the proof is the following. Let us first find an intuitive candidate \underline{K} for the lower bound of the support of ergodic density of K^C (the proof for K^{MC} is the same). Then it is argued that there is a positive probability that the economy experiences a sufficiently long path of "bad" shocks $\{Z_H, Z_H, \dots, Z_H\}$ so that the aggregate capital in this economy falls below $\underline{K} + \eta$ where $\eta > 0$ is an arbitrarily small positive number. At the end I use the Borel-Cantelli lemma again to argue that the probability that $\{Z_H, Z_H, \dots, Z_H\}$ occurs infinitely many times (within the sequence $\{Z_t\}_{t=0}^{+\infty}$) is 1 which implies that the measure of the ergodic distribution of K^C that is accumulated in $(\underline{K}, \underline{K} + \eta)$ is positive.

Let us first notice that the market clearing for "loans" in the economy in which $\mathbb{P}(A_I = 1) = 1$ is:

$$\pi_I K = \left[\frac{1}{1 + \phi Z} \cdot \frac{G(K)}{q_B} + \frac{1}{1 + \phi Z} - 1 \right] \cdot \pi_C \cdot K$$

which implies the following formula for q_B :

$$q_B(K, Z) = \frac{G(K)}{(1 + \phi Z) \frac{\pi_I}{\pi_C} + \phi Z}. \quad (51)$$

Additionally, notice that the formula for the aggregate output of new capital is:

$$I(q_S, K) = \frac{q_S(E, K)}{1 + \phi} \pi_I K$$

which is implied by 2 and the fact that all i-entrepreneurs sell their entire stock of capital when condition 23 holds. Now let us consider a hypothetical economy (which is signed by a subscript H) in which the aggregate output of new capital is:

$$I_H(K) = \frac{q_B(K, Z_H)}{1 + \phi} \pi_I K.$$

Since in equilibrium $q_B(K, Z_H) < q_S(E, K)$ then $I_H(K) < I(q_S, K)$. Let us now derive a more tractable formula for $I_H(K)$:

$$\begin{aligned} I_H(K) &= \frac{q_B(K, Z_H)}{1 + \phi} \pi_I K \\ &= \frac{G(K)}{(1 + \phi Z_H) \frac{\pi_I}{\pi_C} + \phi Z_H} \frac{\pi_I K}{1 + \phi} \\ &= \frac{\alpha A_C (\pi_C / L)^{\alpha-1}}{(1 + \phi Z_H) \frac{\pi_I}{\pi_C} + \phi Z_H} \frac{\pi_I}{1 + \phi} K^\alpha = \Psi K^\alpha \end{aligned}$$

where I have used equation 51, the formula for G and I have defined:

$$\Psi = \frac{\alpha A_C (\pi_C / L)^{\alpha-1}}{(1 + \phi Z_H) \frac{\pi_I}{\pi_C} + \phi Z_H} \frac{\pi_I}{1 + \phi}.$$

Now, it is easy to see that the hypothetical economy is deterministic and has two steady states: the one that is a trivial one with $K_{H,ss} = 0$ and the second with $K_{H,ss}$ that solves:

$$\Psi K^\alpha = \delta K.$$

This means that the non-trivial steady state satisfies $K_{H,ss} = \left(\frac{\Psi}{\delta} \right)^{\frac{1}{1-\alpha}}$. This value becomes a candidate for the lower bound \underline{K} .

Let us come back to the economy in which the output of new capital is $I(q_S, K)$. I will show that for an arbitrarily small positive number $\eta > 0$ there exists a finite number N such that for the real-

ization $\{Z_H, Z_H, \dots, Z_H\}$ of length N the path of the economy's capital stock jumps into neighborhood $(K_{H,ss}, K_{H,ss} + \eta)$.⁴¹ Let us take two arbitrary, positive numbers η_1 and η_2 that satisfy:

$$\eta_1 + \eta_2 = \eta.$$

For $\eta_1 > 0$ let us construct a curve $I_H^{\eta_1}(K) = s(\eta_1) + I_H(K)$ such that $I_H^{\eta_1}(K)$ intersects with δK at $K_{H,ss} + \eta_1$. Suppose that $I_H^{\eta_1}(K)$ characterizes the investment rate in yet another hypothetical economy called economy η_1 . It is obvious that since the aggregate amount of capital in economy η_1 converges to $K_{H,ss} + \eta_1$ then for there exists a finite number of periods N_1 during which economy η_1 that starts at $K \in [K_{H,ss}, \bar{K}^C]$ drops into $(K_{H,ss}, K_{H,ss} + \eta_1 + \eta_2)$.

Now, for each $K \in [K_{H,ss}, \bar{K}^C]$ let us define a number $\tilde{E}(K)$ for which $I(q_S(\tilde{E}(K), K), K) = I_H^{\eta_1}(K)$. This number exists by the continuity of q_S in E (which follows by the bank's FOC combined with equilibrium conditions) and by the fact that $\lim_{E \rightarrow 0} q_S(E, K) = q_B(K, Z_H)$. It is easy to see (again, by the bank's FOC combined with equilibrium conditions) that $\tilde{E}(K)$ is continuous. This means that it attains a minimum for $K \in [K_{H,ss}, \bar{K}^C]$ (a compact set). Let us denote it by K_{min} and by $N_2(K_{min})$ let us denote a natural number that satisfies (by the proof of Proposition 6):

$$\tilde{E}(K_{min}) > \beta^{N_2(K_{min})} \bar{E}.$$

This is clear that the output of new capital in economy that starts with any $K \in [K_{H,ss}, \bar{K}^C]$ and \bar{E} falls below $I_H^{\eta_1}(K)$ if it experiences a sequence $\{Z_H, Z_H, \dots, Z_H\}$ of length $N_2(K_{min})$. Since $I(q_S, K)$ remains below $I_H^{\eta_1}(K)$ if the sequence of "bad" shocks continues then it shrinks and it drops into the region $(K_{H,ss}, K_{H,ss} + \eta_1 + \eta_2)$ faster than the hypothetical economy η_1 . This means that the "true" economy needs at most $N_1 + N_2(K_{min})$ (a finite number) of periods to find itself in $(K_{H,ss}, K_{H,ss} + \eta_1 + \eta_2)$. We set $N = N_1 + N_2(K_{min})$ and notice that $\pi(Z_H)^N$ is a strictly positive number. Now by the Borel-Cantelli lemma we know that the number of sequences $\{Z_H, Z_H, \dots, Z_H\}$ of length N within $\{Z_t\}_{t=0}^{+\infty}$ is infinite with probability 1. Since $\eta > 0$ was an arbitrarily small positive number then we conclude that ergodic density of K is positive in a neighborhood $(K_{H,ss}, K_{H,ss} + \eta)$ of $\underline{K} = K_{H,ss}$. \square

Proposition 8

If $\mathbb{P}(A_I = 1) = 1$ and condition 23 hold then $\frac{d\bar{K}^{MC}}{de}$ evaluated at $\epsilon = 1$ is negative.

Proof. Let us study the limits \bar{K}^C and \bar{E}^C to which the economy with competitive banks converges if the sequence of "good" shocks $\{Z_L, Z_L, \dots, Z_L\}$ is infinite. From the law of motion for capital and from 26 we get that in the limit:

$$\bar{q}_S^C = \frac{\delta(1 + \phi)}{\pi_I} \quad (52)$$

⁴¹I ignore the neighborhood $(K_{H,ss} - \eta, K_{H,ss})$ because if the economy drops into that region then it either converges to $K_{H,ss}$ in a monotone manner (in case of an infinite realization of Z_H which occurs with probability 0 or it jumps above $K_{H,ss}$ and never returns to $(K_{H,ss} - \eta, K_{H,ss})$. Both cases imply that ergodic measure of $(K_{H,ss} - \eta, K_{H,ss})$ is 0.

which means that \bar{q}_S^C is a function of parameters. The market clearing condition for capital ("loans") implies:

$$\pi_I \bar{K}^C = \left[\frac{1}{1 + \phi Z} \cdot \frac{G(\bar{K}^C)}{\bar{q}_B^C} + \frac{1}{1 + \phi Z} - 1 \right] \cdot \pi_C \cdot \bar{K}^C$$

which implies that:

$$\bar{q}_B^C(Z, \bar{K}^C) = \frac{G(\bar{K}^C)}{(1 + \phi Z) \frac{\pi_I}{\pi_C} + \phi Z}. \quad (53)$$

Let us denote $\kappa(Z) = (1 + \phi Z) \frac{\pi_I}{\pi_C} + \phi Z$. Observe that $\bar{k}_F^C = \pi_I \bar{K}^C$ so we can rewrite the bank's FOC as:

$$\begin{aligned} 0 = & \pi(Z_L) (\bar{q}_B^C(Z_L, \bar{K}^C) - \bar{q}_S^C) \cdot (\bar{E}^C + (\bar{q}_B^C(Z_H, \bar{K}^C) - \bar{q}_S^C) \pi_I \bar{K}^C) \\ & + \pi(Z_H) (\bar{q}_B^C(Z_H, \bar{K}^C) - \bar{q}_S^C) \cdot (\bar{E}^C + (\bar{q}_B^C(Z_L, \bar{K}^C) - \bar{q}_S^C) \pi_I \bar{K}^C). \end{aligned} \quad (54)$$

The last equation that characterizes the economy is the law of motion for banks' equity that is derived from the bank's FOC:

$$\bar{E}^C = \beta [\bar{E}^C + (\bar{q}_B^C(Z_L, \bar{K}^C) - \bar{q}_S^C) \pi_I \bar{K}^C] \quad (55)$$

If we plug 53 and 55 into 54 then we can calculate the long-run value of capital:

$$\bar{K}^C = \frac{L}{\pi_C} \left[\frac{\left\{ \pi(Z_L) \frac{\beta}{1-\beta} \frac{1}{\kappa(Z_L)} + \left(\pi(Z_L) + \frac{\pi(Z_H)}{1-\beta} \right) \frac{1}{\kappa(Z_H)} \right\} \alpha A}{\frac{1}{1-\beta} \bar{q}_S^C} \right]^{\frac{1}{1-\alpha}}. \quad (56)$$

Since \bar{q}_S^C is a function of parameters then \bar{K}^C is, too.

Observe that an analogous system of equations can be constructed for monopolistically competitive banks. Equation that corresponds to combination of 54 and 55 in the monopolistic regime is:

$$\begin{aligned} 0 = & \pi(Z_L) \left(\frac{1}{\epsilon} \bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC} \right) \cdot (\bar{E}^{MC} + (\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}) \pi_I \bar{K}^{MC}) \\ & + \pi(Z_H) \left(\frac{1}{\epsilon} \bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC} \right) \cdot (\bar{E}^{MC} + (\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}) \pi_I \bar{K}^{MC}). \end{aligned} \quad (57)$$

Since we can use the "monopolistic" equivalent of equation 53 to eliminate \bar{q}_B^{MC} then it can be concluded that equation 57 defines \bar{K}^{MC} as an implicit function of ϵ (as 57 becomes an equation with one endogenous variable). I use this fact together with the Implicit Function Theorem to check the sign of $\frac{d\bar{K}^{MC}}{d\epsilon}$ evaluated at $\epsilon = 1$ and $\bar{K}^{MC} = \bar{K}^C$.

Let us define $F(\bar{K}^{MC}, \epsilon)$ as the RHS of the equation above. I calculate (after plugging $\bar{E}^C = \bar{E}^{MC}$ from 55):

$$\begin{aligned} F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1) = & \frac{\beta \pi_L}{1 - \beta} \cdot \bar{q}_{B,K}^{MC}(Z_L, \bar{K}^{MC}) \cdot (\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}) \\ & + \pi_L \cdot \bar{q}_{B,K}^{MC}(Z_L, \bar{K}^{MC}) \cdot (\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}) \end{aligned}$$

$$\begin{aligned}
& + \frac{\beta\pi_L}{1-\beta} \cdot \bar{q}_{B,K}^{MC}(Z_L, \bar{K}^{MC}) \cdot (\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}) \\
& + \pi_L \cdot \bar{q}_{B,K}^{MC}(Z_H, \bar{K}^{MC}) \cdot (\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}) \\
& + \frac{\pi_H \cdot \bar{q}_{B,K}^{MC}(Z_H, \bar{K}^{MC})}{1-\beta} \cdot (\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}) + \frac{\pi_H \cdot \bar{q}_{B,K}^{MC}(Z_L, \bar{K}^{MC})}{1-\beta} \cdot (\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}),
\end{aligned}$$

observe that by 53 \bar{q}_B^{MC} is a function of \bar{K}^{MC} and hence $\bar{q}_{B,K}^{MC}$ denotes the derivative with respect to \bar{K}^{MC} . Note that $\bar{q}_S^{MC} = \bar{q}_S^C$. We use 53, formula for $G(\cdot)$ and the definition of $\kappa(Z)$ to obtain:

$$\begin{aligned}
F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1) &= G'(\bar{K}^{MC}) \cdot \left\{ \left(\frac{\pi(Z_H)}{1-\beta} + \pi(Z_L) \right) \right. \\
&\quad \cdot \left[\frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}}{\kappa(Z_L)} + \frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}}{\kappa(Z_H)} \right] + \frac{2\pi(Z_L)\beta}{1-\beta} \cdot \frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}}{\kappa(Z_L)} \Big\}
\end{aligned}$$

where I have used the fact that $\bar{q}_{B,K}^{MC} = G'(\bar{K}^{MC}) \frac{1}{\kappa(Z)}$ (see equation 53). Observe that from 54 and 55 we get:

$$\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C = - \left(\frac{1-\beta}{\beta} + \frac{1-\pi(Z_L)}{\beta\pi(Z_L)} \right) [\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C]. \quad (58)$$

This relationship implies that:

$$\begin{aligned}
& \left(\frac{\pi(Z_H)}{1-\beta} + \pi(Z_L) \right) \frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)} + \frac{2\pi(Z_L)\beta}{1-\beta} \cdot \frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)} \\
& = - \left[\frac{\pi(Z_H)}{1-\beta} + \pi(Z_L) \right] \frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)} > 0
\end{aligned}$$

because we know that in equilibrium $\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC} > 0$. Plugging back to the expression for $F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1)$ yields:

$$\begin{aligned}
F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1) &= G'(\bar{K}^{MC}) \cdot \left(\frac{\pi(Z_H)}{1-\beta} + \pi(Z_L) \right) \\
&\quad \cdot \left[\frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_H)} - \frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)} \right]
\end{aligned}$$

It is clear that since $G'(\bar{K}^{MC}) < 0$, $\frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^{MC}}{\kappa(Z_H)} > 0$, $\frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^{MC}}{\kappa(Z_L)} < 0$ then $F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1) < 0$. Let us consider $F_\epsilon(\bar{K}^{MC}, \epsilon = 1)$ now:

$$\begin{aligned}
F_\epsilon(\bar{K}^{MC}, \epsilon = 1) &= -\frac{1}{\epsilon^2} \cdot \left\{ \frac{\pi(Z_L)\beta}{1-\beta} \bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) \cdot (\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C) + \pi(Z_L) \bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) \cdot (\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C) \right. \\
&\quad \left. + \frac{\pi(Z_H)}{1-\beta} \bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) \cdot (\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C) \right\}.
\end{aligned}$$

Let us use 58 again to calculate:

$$\begin{aligned} & \frac{\beta}{1-\beta} \cdot (\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C) + (\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C) \\ &= -(\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C) \left[\frac{\pi(Z_H)}{\pi(Z_L)(1-\beta)} \right] > 0. \end{aligned}$$

Plugging back to the formula for $F_\epsilon(\bar{K}^{MC}, \epsilon = 1)$ gives us:

$$\begin{aligned} F_\epsilon(\bar{K}^{MC}, \epsilon = 1) &= \frac{1}{\epsilon^2} \frac{\pi(Z_H)}{1-\beta} G(\bar{K}^{MC}) \\ &\cdot \left[-\frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_H)} + \frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)} \right] \end{aligned}$$

where I have used the formula 53. Since $\frac{\bar{q}_B^{MC}(Z_L, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_H)} > 0$, $\frac{\bar{q}_B^{MC}(Z_H, \bar{K}^{MC}) - \bar{q}_S^C}{\kappa(Z_L)} < 0$ then $F_\epsilon(\bar{K}^{MC}, \epsilon = 1) < 0$. By the Implicit Function Theorem we get the following result:

$$\frac{d\bar{K}^{MC}}{d\epsilon} = -\frac{F_\epsilon(\bar{K}^{MC}, \epsilon = 1)}{F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1)} < 0.$$

This completes the proof.⁴² It is useful, however, to compute a more precise expression for $\frac{d\bar{K}^{MC}}{d\epsilon}$ (it will be useful to prove next propositions):

$$\begin{aligned} \frac{d\bar{K}^{MC}}{d\epsilon} &= -\frac{F_\epsilon(\bar{K}^{MC}, \epsilon = 1)}{F_{\bar{K}^{MC}}(\bar{K}^{MC}, \epsilon = 1)} \\ &= -\frac{1}{\epsilon^2} \frac{\frac{\pi(Z_H)}{1-\beta}}{\pi(Z_L) + \frac{\pi(Z_H)}{1-\beta}} \frac{1}{1-\alpha} \bar{K}^{MC} \end{aligned}$$

□

Proposition 9

If $\pi(Z_L)\beta > \alpha$, $\mathbb{P}(A_I = 1) = 1$ and condition 23 hold then $\frac{d\bar{E}^{MC}}{d\epsilon}$ evaluated at $\epsilon = 1$ is positive.

Proof. Let us observe that by 55, the long run value of bank's equity can be rewritten as:

$$\bar{E}^C = \frac{\beta}{1-\beta} (\bar{q}_B^C(\bar{K}^C, Z_L) - \bar{q}_S) \pi_I \bar{K}^C$$

⁴²Observe that I have not shown that there is an infinite number of trajectories that approach \bar{K}^C for $\{Z_t\}_{t=1}^{+\infty}$. Analytic proof of this fact (like it was in the case for the lower bounds) is much harder to construct so I have used a numerical verification to show that the trajectory that corresponds to a sufficiently long path of $\{Z_L, Z_L, \dots, Z_L\}$ converges to \bar{K}^C . Then by a similar argument (i.e., the Borel-Cantelli lemma) one can argue that the mass of the ergodic distribution in the neighborhood of \bar{K}^C is positive.

where \bar{q}_S is a function of parameters. I use 53 to reformulate the equation above:

$$\bar{E}^C = \frac{\beta\pi_I}{1-\beta} \left(\frac{\alpha A_C L^{1-\alpha} \pi_C^{\alpha-1}}{\kappa(Z_L)} (\bar{K}^C)^\alpha - \bar{q}_S \bar{K}^C \right).$$

This defines \bar{E}^C as a strictly concave function of \bar{K}^C . This function attains its maximum at:

$$\bar{K}_E^C = \frac{L}{\pi_C} \left(\frac{\alpha^2 A_C}{\bar{q}_S \kappa(Z_L)} \right)^{\frac{1}{1-\alpha}}$$

and it decreases for $\bar{K}^C > \bar{K}_E^C$. This inequality holds in our case. It is because (from 56):

$$\bar{K}^C = \frac{L}{\pi_C} \left[\frac{\left\{ \pi(Z_L) \frac{\beta}{1-\beta} \frac{1}{\kappa(Z_L)} + \left(\pi(Z_L) + \frac{\pi(Z_H)}{1-\beta} \right) \frac{1}{\kappa(Z_H)} \right\} \alpha A}{\frac{1}{1-\beta} \bar{q}_S} \right]^{\frac{1}{1-\alpha}} > \frac{L}{\pi_C} \left(\frac{\alpha^2 A_C}{\bar{q}_S \kappa(Z_L)} \right)^{\frac{1}{1-\alpha}}$$

which is equivalent to:

$$\pi(Z_L)\beta + [\pi(Z_L)(1-\beta) + \pi(Z_H)] \frac{\kappa(Z_L)}{\kappa(Z_H)} > \alpha$$

and since by assumption $\pi(Z_L)\beta > \alpha$ then the inequality above follows. Let us use Proposition 8: if ϵ increases then \bar{K}^{MC} drops. Since the value of \bar{K}^{MC} that corresponds to $\epsilon = 1$ satisfies $\bar{K}^{MC} = \bar{K}^C > \bar{K}_E^C$ and since \bar{E}^C is strictly concave in \bar{K}^{MC} then if \bar{K}^{MC} drops in response to growth in ϵ then \bar{E}^{MC} grows. \square

Proposition 10

If $\pi(Z_L)\beta > \alpha, \mathbb{P}(A_I = 1) = 1$ and condition 23 hold then $\left(\frac{d\bar{K}^{MC}}{d\epsilon} \right)_{\bar{K}^{MC}}$ (evaluated at $\epsilon = 1$) does not depend on Z_H .

Proof. This proof is immediate from what was shown in the proof of Proposition 8. Since:

$$\frac{d\bar{K}^{MC}}{d\epsilon} = -\frac{1}{\epsilon^2} \frac{\frac{\pi(Z_H)}{1-\beta}}{\pi(Z_L) + \frac{\pi(Z_H)}{1-\beta}} \frac{1}{1-\alpha} \bar{K}^{MC}$$

then indeed $\left(\frac{d\bar{K}^{MC}}{d\epsilon} \right)_{\bar{K}^{MC}}$ (evaluated at $\epsilon = 1$) does not depend on Z_H , as:

$$\left(\frac{d\bar{K}^{MC}}{d\epsilon} \right)_{\bar{K}^{MC}} = -\frac{1}{\epsilon^2} \frac{\frac{\pi(Z_H)}{1-\beta}}{\pi(Z_L) + \frac{\pi(Z_H)}{1-\beta}} \frac{1}{1-\alpha}$$

\square

Proposition 11

If $\pi(Z_L)\beta > \alpha, \mathbb{P}(A_I = 1) = 1$ and condition 23 hold then $\frac{(\frac{d\bar{E}^{MC}}{d\epsilon})}{\bar{E}^{MC}}$ (evaluated at $\epsilon = 1$) decreases in Z_H .

Proof. First recall (from the proof of Proposition 9) that:

$$\bar{E}^C = \frac{\beta\pi_I}{1-\beta} \left(\frac{\alpha A_C L^{1-\alpha} \pi_C^{\alpha-1}}{\kappa(Z_L)} (\bar{K}^C)^\alpha - \bar{q}_S \bar{K}^C \right).$$

Since we want to calculate the expression $\frac{(\frac{d\bar{E}^{MC}}{d\epsilon})}{\bar{E}^{MC}}$ for $\epsilon = 1$ then $\bar{E}^C = \bar{E}^{MC}$. Observe that:

$$\frac{d\bar{E}^{MC}}{d\epsilon} = \frac{\beta\pi_I}{1-\beta} \left(\Psi \cdot \alpha \cdot (\bar{K}^C)^{\alpha-1} - \bar{q}_S \right) \cdot \frac{d\bar{K}^{MC}}{d\epsilon}$$

where $\Psi = \frac{\alpha A_C L^{1-\alpha} \pi_C^{\alpha-1}}{\kappa(Z_L)}$ and since I evaluate all terms for $\epsilon = 1$ then $\bar{K}^C = \bar{K}^{MC}$. Let us calculate $\frac{(\frac{d\bar{E}^{MC}}{d\epsilon})}{\bar{E}^{MC}}$ now:

$$\begin{aligned} \frac{(\frac{d\bar{E}^{MC}}{d\epsilon})}{\bar{E}^{MC}} &= \frac{\frac{\beta\pi_I}{1-\beta} \left(\Psi \cdot \alpha \cdot (\bar{K}^{MC})^{\alpha-1} - \bar{q}_S \right) \cdot \frac{d\bar{K}^{MC}}{d\epsilon}}{\frac{\beta\pi_I}{1-\beta} \left(\Psi \cdot (\bar{K}^{MC})^\alpha - \bar{q}_S \bar{K}^{MC} \right)} \\ &= \frac{\Psi \cdot \alpha - \bar{q}_S (\bar{K}^{MC})^{1-\alpha}}{\Psi - \bar{q}_S (\bar{K}^{MC})^{1-\alpha}} \cdot \left(\frac{d\bar{K}^{MC}}{d\epsilon} \right). \end{aligned}$$

Let us study the sign of the derivative of the expression above with respect to Z_H . First recall that $\frac{d\bar{K}^{MC}}{d\epsilon}$ does not depend on Z_H (by Proposition 10) and hence it is treated as a constant in further calculations. Observe that Ψ , \bar{q}_S and \bar{K}^{MC} all depend on Z_H (see the formula for \bar{K}^{MC} in equation 56, 52, $\Psi = \frac{\alpha A_C L^{1-\alpha} \pi_C^{\alpha-1}}{\kappa(Z_L)}$ and recall the definition of $\kappa(Z) = (1 + \phi Z) \frac{\pi_I}{\pi_C} + \phi Z$ and the fact that $\phi = \frac{1-\beta}{\beta(\pi_I + \pi_C(\pi(Z_L)Z_L + \pi(Z_H)Z_H))}$). This means that:

$$\begin{aligned} \left(\frac{(\frac{d\bar{E}^{MC}}{d\epsilon})}{\bar{E}^{MC}} \right)'_{Z_H} &= \left(\frac{d\bar{K}^{MC}}{d\epsilon} \right) \cdot (1 - \alpha) \\ &\cdot \left(\frac{\Psi'_{Z_H} \cdot \bar{q}_S (\bar{K}^{MC})^{1-\alpha} - \bar{q}'_{S,Z_H} (\bar{K}^{MC})^{1-\alpha} \Psi - \bar{q}_S (1 - \alpha) (\bar{K}^{MC})^{-\alpha} \Psi \left(\frac{d\bar{K}^{MC}}{dZ_H} \right)}{(\Psi - \bar{q}_S (\bar{K}^{MC})^{1-\alpha})^2} \right). \end{aligned} \quad (59)$$

Now it is easy to verify that: $\bar{q}'_{S,Z_H} < 0$ and $\Psi'_{Z_H} > 0$. Let us concentrate on the sign of $\frac{d\bar{K}^{MC}}{dZ_H}$. It is hard to calculate this derivative directly from the formula 56 as it contains \bar{q}_S , $\kappa(Z_H)$ and $\kappa(Z_L)$ that are themselves functions of Z_H . Let us do it in a different way, instead. Suppose that economies with two different levels \bar{Z}_H and \underline{Z}_H (where $\bar{Z}_H > \underline{Z}_H$) satisfy $\bar{K}^C(\bar{Z}_H) > \bar{K}^C(\underline{Z}_H)$ and that they start at the same initial values of K and E , i.e., $K = \bar{K}^C(\underline{Z}_H)$ and $E = \bar{E}^C(\underline{Z}_H)$. The

economy characterized by \underline{Z}_H exhibits the level of output of new capital $I_{\underline{Z}_H}(\bar{K}^C(\underline{Z}_H), \bar{E}^C(\underline{Z}_H))$ that preserves the current level of capital (i.e., $K'(\underline{Z}_H) = \bar{K}^C(\underline{Z}_H)$ - which follows by the definition of the upper bound of ergodic distributions - $\bar{K}^C(\underline{Z}_H)$ and $\bar{E}^C(\underline{Z}_H)$). Let us investigate the amount of new capital associated with economy described by \bar{Z}_H . From our assumption we have $\bar{K}^C(\bar{Z}_H) > \bar{K}^C(\underline{Z}_H)$ and hence $K'(\bar{Z}_H) > K'(\underline{Z}_H)$. This in turn implies that:

$$I_{\bar{Z}_H}(\bar{K}^C(\underline{Z}_H), \bar{E}^C(\underline{Z}_H)) > I_{\underline{Z}_H}(\bar{K}^C(\underline{Z}_H), \bar{E}^C(\underline{Z}_H)). \quad (60)$$

On the other hand, however, equation 54 (with $Z_H = \underline{Z}_H$ and $Z_H = \bar{Z}_H$, respectively) indicates that:

$$q_{S, \underline{Z}_H} > q_{S, \bar{Z}_H}.$$

It is because increase in Z_H leads to a decrease in $q_B(Z_H)$ (equation 53) which means that (by the Implicit Function Theorem and the proof of Theorem 1) q_S drops.⁴³ This implies that:

$$I_{\bar{Z}_H}(\bar{K}^C(\underline{Z}_H), \bar{E}^C(\underline{Z}_H)) < I_{\underline{Z}_H}(\bar{K}^C(\underline{Z}_H), \bar{E}^C(\underline{Z}_H))$$

that contradicts 60. This in turn means that $\bar{K}^C(\bar{Z}_H) < \bar{K}^C(\underline{Z}_H)$ which is a contradiction. This means that (differentiability of \bar{K}^C with respect to Z_H is assumed) that:

$$\frac{d\bar{K}^{MC}}{dZ_H} < 0.$$

This all means that is negative (it is because $\frac{d\bar{K}^{MC}}{dZ_H} < 0$).

□

⁴³Note that changes in Z_H have an indirect impact on $q_B(Z_L)$ through ϕ . This influence, though, has a second-order influence on q_S in comparison to $q_B(Z_H)$.

2 Frictional and Keynesian unemployment in European economies

Abstract

Knowledge of the unemployment structure (that consists of e.g. frictional and Keynesian unemployment) is necessary for the policymakers to fight it effectively. The problem is that these components are not directly observable. This paper develops the unemployment decomposition method that is based on the DSGE model with two frictions (standard search frictions in the labor market and in the market for products) and price stickiness that allows for distinction between frictional and Keynesian unemployment. The model is used to study the structure of unemployment in four largest economies in the Eurozone: Germany, France, Italy and Spain.

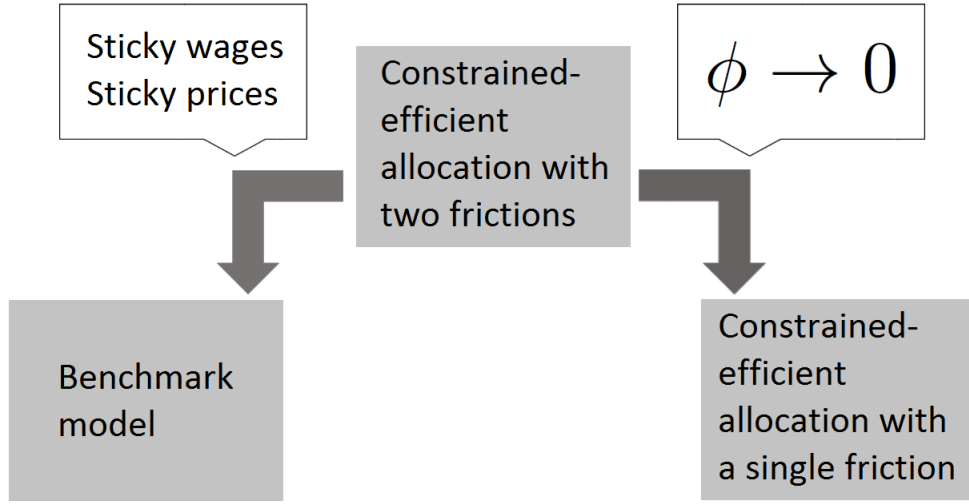
2.1 Introduction

It is well-understood that since the structure of unemployment is not homogenous, the policies that aim at decreasing unemployment should be adjusted to its specific heterogeneity. For the unemployment's components (like Keynesian or frictional unemployment) are not directly observable, there is a need for a theoretical method that decomposes the recorded time series of unemployment. I develop a framework that allows for such decomposition: I add two search frictions and price/wage rigidities into otherwise standard RBC model. I use this construction to analyze the unemployment structure in Germany, France, Italy and Spain.

[Michaillat and Saez (2015)] have recently shown that models with frictions in both labor and product markets can be used to decompose total unemployment into three components: Keynesian, classical and frictional unemployment. They develop a theoretical, continuous-time model with search frictions both in the market for goods and the labor market, use their model to conduct a comparative-statics analysis and study the sources of labor market fluctuations in the US. They highlight the role of sticky wages and sticky prices in the propagation of shocks: with fixed prices, a drop in aggregate demand decreases product market tightness (the ratio of demand on products and manufacturer's capacity), which lowers sales made by producers and increases the idle time of hired employees. Since workers remain idle a larger proportion of the time, they become less profitable to employers, and the demand for labor decreases. The drop in labor demand raises unemployment. With flexible prices, a decrease in demand causes a decline of price level and hence it is absorbed, so it does not affect either product market tightness or unemployment. This analysis can be seen as an attempt to incorporate the mechanism described by Michaillat and Saez into otherwise standard RBC framework.

[Michaillat (2012)] has conducted a decomposition of unemployment for the US economy and has distinguished two main components: rationing unemployment and frictional unemployment. Rationing unemployment emerges in the Mortensen-Pissarides framework used by Michaillat when wages remain above the marketclearing level and its source is the combination of diminishing marginal returns to

Figure 11: Decomposition method - intuition



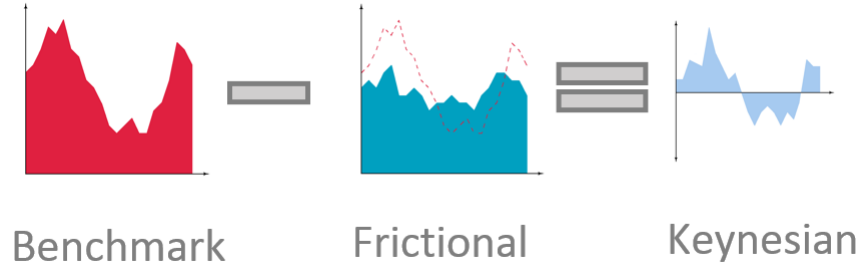
labor and wage stickiness. Keynesian unemployment that is present in my analysis bears some conceptual similarities to those of rationing unemployment, but their source is different: I assume constant returns to scale and Keynesian unemployment arises as a result of three factors: price stickiness, wage stickiness and frictions in the market for goods. My analysis is conducted in the standard DSGE framework (contrary to the Mortensen-Pissarides model of labor market used by Michaillat) and therefore allows for many potential extensions: e.g., studying fiscal and monetary policy.

My work is related to [Bai et al. (2011)], who show that demand shocks are responsible for the TFP volatility if the product market frictions are in place. However, they abstract from frictions in the labor market and from price rigidities which are present in our model and give rise to our decomposition method.

The decomposition method presented in this work is based on the following intuitions: the presence of recruitment/training costs, the fact that the hiring process is time-consuming and that the mismatch of qualifications imply that some workers remain unemployed even if they actively search for jobs. This gives rise to frictional unemployment. The frictions that cause frictional unemployment are captured by the Diamond-Mortensen-Pissarides framework. Additionally, if prices and wages are sticky then the labor market may cease to clear. On the top of that, changes in aggregate demand influence the probability of selling manufactured goods and thus make firms adjust their workforce. Price rigidities and changes in aggregate demand give rise to Keynesian unemployment.

To disentangle the two types of unemployment, I construct three DSGE models that are related to each other in the following way (which is illustrated in Figure 11). First, in Section 2.2, I consider a competitive economy with two frictional markets: product market and labor market. It is a more general version of the benchmark model where no additional assumptions about prices and wages are made. Second, in section 2.3, I consider a constrained-efficient economy with two frictions and

Figure 12: Unemployment accounting and the decomposition method



derive prices that make the allocation discussed in Section 2.2 equivalent to the planner's solution in the non-stochastic steady state. Third, I analyze economy with a single friction that is present in the labor market (Section 2.4). In the same section, I prove that the "limit" of constrained-efficient economies with two frictions (when the friction described by parameter $\phi > 0$ in the product market decays $\phi \rightarrow 0$) is the economy with a single friction (which is summarized by Theorem 3). Fifth, I calibrate and estimate the benchmark model to match empirical data and then I use the Kalman filter to extract paths of stochastic shocks that adjust the model to the observed time series. Finally, I use the extracted shocks to run the model with a single friction and I calculate the corresponding path of unemployment. It is called "frictional" unemployment as it is associated with the model where the only friction is the search friction in the labor market. By these considerations, the difference between the unemployment rate observed in the data and the "frictional" unemployment can be attributed solely to two factors: friction in the market for products (that gives rise to significance of aggregate demand) and price/wage stickiness. It is therefore called "Keynesian" unemployment. The intuition behind this accounting method is presented in Figure 12.

2.2 Competitive allocation with two frictions

2.2.1 Households

The model is populated by identical, infinitely-lived households (workers) of measure one. Similarly to [Bai et al. (2011)], they have to exert effort to purchase consumption goods. This process is modeled in the following manner: households visit manufacturers to buy goods. A single worker makes v visits and each of them is successful (i.e., results in a purchase of a unit of consumption good) with probability $q_G(\theta_G)$, where θ_G is the tightness of the product market (which is defined later). This implies, that the total number of purchased goods $q_G(\theta_G)v$ is related to consumption in the following way:

$$c = q_G(\theta_G)v. \quad (61)$$

I abstract from randomness at individual level - this means that every household makes $q_G(\theta_G)v$ successful visits and hence households' consumption levels and incomes are identical.⁴⁴ There is a

⁴⁴I abstract from randomness for individual firms, too.

utility cost of making v visits that is captured by a convex function $G(v) > 0$ (more specifically, I consider $G(v) = \frac{\phi}{2}v^2$ where $\phi > 0$). There are two stochastic, Markovian disturbances that affect the economy: the first affects consumer's demand - a_d and the second influences the productivity level of firms - a_z . By N_{-1} I denote the fraction of workers that were employed at the end of the previous period. Each worker derives utility $\exp(a_d) \cdot u(c)$ from goods consumed in the current period where u is twice differentiable and strictly concave. Worker's income consists of: nominal labor income wN , where w is wage expressed in terms of price of shares and income from selling shares s together with dividends Πs associated with firms' profits Π . Household uses its income to purchase shares that can be sold in the following period s' and to buy consumption goods.⁴⁵ Let us denote the set of state variables by $Z = \{a_d, a_z, N_{-1}\}$ and let $a = \{a_d, a_z\}$ be a vector of exogenous states. It means that the dynamic problem of a worker can be described by the following Bellman equation:

$$W(s, Z) = \max_{c, v, s'} \{ \exp(a_d) \cdot u(c) - G(v) + \beta \mathbb{E}_{a'|a} W(s', Z') \} \quad (62)$$

subject to :

$$c = q_G(\theta_G(Z))v,$$

$$p(Z) \cdot c + s' = s(1 + \Pi) + w(Z) \cdot N,$$

$$N = N(Z),$$

where by p I denote the price of consumption goods, θ_G , p , w are taken by workers as given.⁴⁶ The second constraint is consumer's budget constraint and the third one is the perceived law of motion of endogenous state variable.⁴⁷ Let us eliminate c and s' from the maximization problem. I substitute c from 61 into the budget constraint and into Bellman equation. Then I plug s' from the budget constraint into Bellman equation and I derive the FOC with respect to v :

$$\exp(a_d)u'(c)q_G(\theta_G) - G'(v) = pq_G(\theta_G)\beta\mathbb{E}_{a'|a}W_s(s', Z') \quad (63)$$

The associated envelope condition is:

$$W_s(s, Z) = (1 + \Pi) \cdot \beta \mathbb{E}_{a'|a} W_s(s', Z'). \quad (64)$$

I calculate $\beta \mathbb{E}_{a'|a} W_s(s', Z')$ from 63 and plug into 64 to get the formula for $W_s(s, Z)$. I take this expression and plug it back into 63 to obtain the Euler equation:

$$1 = \mathbb{E}_{a'|a} \left(\left\{ \beta \frac{p \cdot q_G(\theta_G)}{p' \cdot q_G(\theta'_G)} \left[\frac{q_G(\theta'_G) \exp(a'_d) \cdot u'(c') - G'(v')}{q_G(\theta_G) \exp(a_d) \cdot u'(c) - G'(v)} \right] \right\} \cdot (1 + \Pi') \right). \quad (65)$$

⁴⁵I use primes to denote forward lags of variables

⁴⁶I suppress the arguments of pricing functions w and p and function θ_G to simplify notation.

⁴⁷I.e., it captures the implicit assumption about workers' rational expectations.

2.2.2 Firms

There is measure one of identical firms. They are owned by households and use labor as the only input. Their production function is linear in labor and is affected by multiplicative productivity shocks $\exp(a_z)$.⁴⁸ Since there are search frictions present in the market for products, firms do not sell their entire output - they sell only a proportion $f_G(\theta_G)$ of it.⁴⁹ I assume that job destruction takes place at the beginning of period so that the number of workers that remain in the workforce at the beginning of the current period is $(1 - \sigma)n_{-1}$, where $0 < \sigma < 1$ is exogenous separation rate of worker-employer relationship and n_{-1} is number of workers hired in the firm at the end of the previous period. To recruit workers, firms post vacancies v_L which are filled with probability $q_L(\theta_L)$ where θ_L is labor market tightness. A single vacancy costs κ units of firm's production capacity. Firms pay wages w to workers they hire. This means that firm's problem can be easily summarized by the following Bellman equation:

$$\begin{aligned} J(n_{-1}, Z) = & \max_{v_L, n \in [0, 1]} \{p(Z) f_G(\theta_G(Z)) [\exp(a_z)n - \kappa v_L] \\ & - w(Z)n + \beta \mathbb{E}_{a'|a} \Delta(Z', Z) \cdot J(n, Z')\}, \\ & \text{subject to :} \\ & n = (1 - \sigma)n_{-1} + q_L(\theta_L(Z)) \cdot v_L, \\ & N = N(Z), \end{aligned}$$

where $\beta \Delta(Z', Z)$ is the factor at which firms discount future profits and $\Delta(Z', Z)$ is defined as follows:

$$\Delta(Z', Z) = \frac{p \cdot q_G(\theta_G)}{p' \cdot q_G(\theta'_G)} \left[\frac{q_G(\theta'_G) \exp(a'_d) \cdot u'(c') - G(v')}{q_G(\theta_G) \exp(a_d) \cdot u'(c) - G(v)} \right]. \quad (66)$$

Firms take prices $p(Z)$, wages $w(Z)$, tightnesses $\theta_G(Z)$, $\theta_L(Z)$ and the discount factor $\Delta(Z', Z)$ as given. Firms's FOC with respect to v_L is:

$$\begin{aligned} p(Z) \cdot f_G(\theta_G) [\exp(a_z)q_L(\theta_L) - \kappa] - w(Z) \cdot q_L(\theta_L) \\ + q_L(\theta_L) \beta \mathbb{E}_{a'|a} \Delta(Z', Z) \cdot J_n(n, Z') = 0 \end{aligned} \quad (67)$$

and the envelope condition reads:

$$\begin{aligned} J_n(n_{-1}, Z) = & p f_G(\theta_G) \exp(a_z)(1 - \sigma) - w(1 - \sigma) \\ & + \beta \mathbb{E}_{a'|a} \Delta(Z', Z) \cdot J_n(n, Z')(1 - \sigma). \end{aligned} \quad (68)$$

⁴⁸I assume the linearity to avoid job rationing described by [Michaillat (2012)] so that Keynesian unemployment in my model can be explained solely by the presence of frictions in the product market and price/wage stickiness.

⁴⁹It is then assumed that the remaining proportion of output is wasted.

Firm's current profit is given by:

$$\Pi = p(Z) f_G(\theta_G(Z)) [\exp(a_z)n - \kappa v_L] - w(Z) \cdot n. \quad (69)$$

2.2.3 Law of motion in the labor market and consistency conditions

The law of motion for employment is:

$$N = (1 - \sigma)N_{-1} + M^L(U, v_L), \quad (70)$$

where M^L is the matching function and $U = 1 - (1 - \sigma)N_{-1}$ denotes the aggregate number of unemployed workers. A similar concept is present in the market for goods: there is a number of $M^G(v, \exp(a_z)N - \kappa v_L)$ successful trades given the number of visits v chosen by households and the total amount of goods supplied by firms is:

$$T = \exp(a_z)N - \kappa v_L. \quad (71)$$

Tightness in the labor market and tightness in the market for goods are defined as follows:

$$\theta_L = \frac{1 - (1 - \sigma)N_{-1}}{v_L}, \quad (72)$$

$$\theta_G = \frac{T}{v}. \quad (73)$$

I consider the following specifications of M^G and M^L :

$$M^G(v, T) = \frac{vT}{(v^{\alpha_G} + T^{\alpha_G})^{\frac{1}{\alpha_G}}},$$

$$M^L(U, v_L) = \frac{v_L U}{(v_L^{\alpha_L} + U^{\alpha_L})^{\frac{1}{\alpha_L}}},$$

where $\alpha_G > 1$ and $\alpha_L > 1$. These specifications of matching functions were introduced by [Den Haan et al. (2000)] and I use them because they are convenient from the perspective of the decomposition exercise.⁵⁰ Since both M^L and M^G are specified as constant returns to scale functions, then probabilities q_L , q_G , f_G can be expressed as functions of tightness that corresponds to a given market.⁵¹ Values q_L , q_G , f_G satisfy:

$$q_L = \frac{M^L}{v_L},$$

$$q_G = \frac{M^G}{v},$$

⁵⁰I do not use another common specification - the Cobb-Douglas function - as the arrival rates of offers in markets (e.g., q_L) may exceed 1. Matching function presented by [Den Haan et al. (2000)] standardizes these rates as numbers from interval $[0, 1]$ which is crucial for my decomposition method.

⁵¹I suppress the arguments of functions q_L , q_G , f_G to economize on notation.

$$f_G = \frac{M^G}{T}.$$

Additionally, individual decisions of firms are consistent with aggregate employment:

$$n = N.$$

I impose market clearing condition for the asset market:

$$s = 1. \tag{74}$$

The resource constraint for the analyzed economy is:

$$c = f_G(\theta_G)T. \tag{75}$$

Stochastic disturbances are described by the following autoregressive processes:

$$a_d' = \rho_D a_d + \epsilon_d', \tag{76}$$

$$a_z' = \rho_Z a_z + \epsilon_z', \tag{77}$$

where $0 < \rho_Z, \rho_D < 1$ and $\begin{bmatrix} \epsilon_d \\ \epsilon_z \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Sigma_{2 \times 2}\right)$, where $\Sigma_{2 \times 2}$ is variance-covariance matrix. Equations 63 and 65-77 constitute a system of 15 equations that contains 17 variables. This means that values of prices and wages have to be pinned down by two additional conditions - a situation that emerges naturally as a consequence of the presence of frictions in markets. I derive the two remaining equations by assuming that competitive equilibrium shares its steady state allocation with the constrained-efficient outcome (i.e. it is an allocation that is identical with the planner's solution given two constraints: frictions in the labor market and frictions in the market for products).

2.3 Optimal allocation with two frictions and price-setting/wage-setting formulas

In this section I compute the planner's solution that corresponds to the decentralized economy with frictional labor and product markets discussed above. Planner's problem can be summarized by the following Bellman equation:

$$V(Z) = \max_{c, v_L, v, N} \{ \exp(a_d) \cdot u(c) - G(v) + \beta \mathbb{E}_{a'|a} V(Z') \}$$

subject to :

$$c = M^G(v, \exp(a_z)N - \kappa v_L), \quad (78)$$

$$N = (1 - \sigma)N_{-1} + M^L(1 - (1 - \sigma)N_{-1}, v_L), \quad (79)$$

where V is the value function associated with the planner's problem. First order conditions are:⁵²

$$M_v^G \cdot \exp(a_d) \cdot u'(c) = G'(v), \quad (80)$$

$$\beta \mathbb{E}_{a'|a} V_N(Z') \cdot M_{v_L}^L + \exp(a_d) \cdot u'(c) \cdot M_T^G \cdot (\exp(a_z)M_{v_L}^L - \kappa) = 0. \quad (81)$$

The envelope condition is:

$$V_N(Z) = \{\exp(a_d) \cdot u'(c) \cdot \exp(a_z) \cdot M_T^G + \beta \mathbb{E}_{a'|a} V_N(Z')\} (1 - \sigma) [1 - M_U^L]. \quad (82)$$

Equations 78-82 together with 72, 73, 76 and 77 characterize the planner's solution.

The following proposition presents formulas for prices and wages which guarantee that competitive equilibrium allocation from Section 2.2 has the same steady state as the planner's solution discussed above.⁵³

Proposition 12. *If the steady state value of price p is given by*

$$p = \frac{1}{1 + \theta_G^{\alpha_G}} \cdot \frac{u'(c)}{\beta W_s} \quad (83)$$

and wage w is characterized by the system:

$$\begin{cases} w = \beta \left[f_G(1 - \sigma) + (\beta(1 - \sigma) - 1) \cdot \frac{1}{u'(c)} \cdot \frac{f_G}{M_T^G} \cdot \frac{(1 - \frac{\kappa}{q_L})}{(1 - \frac{\kappa}{M_{v_L}^L})} \cdot V_N \right] \cdot p, \\ V_N = \frac{u'(c) \cdot M_T^G (1 - \sigma) [1 - M_U^L]}{1 + (1 - \sigma) [1 - M_U^L]}, \end{cases} \quad (84)$$

then the competitive allocation has the same steady state as the constrained-efficient outcome.

Proof. My strategy is to show, that the steady state allocation determined by equations 63-77, 83 and 84 satisfies conditions that characterize planner's solution. It is immediate that equations 78-79, 82, conditions that characterize θ_L , θ_G and shocks appear both in the system that characterizes competitive outcome and in the system that describes the optimal allocation. It means that it remains to show that conditions that characterize the decentralized outcome imply 80 and 81 (observe that planner's envelope condition is equivalent to the second equation of 84).

⁵²FOCs are derived with respect to v and v_L after substitution of c and N' from the constraints.

⁵³All variables in Proposition 12 take their steady state values.

Let us begin with equation 80. Observe that from 63:

$$u'(c)q_G(\theta_G) - G'(v) = pq_G(\theta_G)\beta W_s$$

and from the formula for price: $\frac{1}{1+\theta_G^{\alpha_G}} \cdot \frac{u'(c)}{\beta W_s}$ I get:⁵⁴

$$M_v^G \cdot u'(c) = G'(v),$$

which is identical to the steady state version of 80.

I derive 81 from conditions that describe the competitive allocation. The first equation that characterizes wages is:

$$w = \beta \left[f_G(1 - \sigma) + (\beta(1 - \sigma) - 1) \cdot \frac{1}{u'(c)} \cdot \frac{f_G}{M_T^G} \cdot \frac{\left(1 - \frac{\kappa}{q_L}\right)}{\left(1 - \frac{\kappa}{M_{v_L}^L}\right)} \cdot V_N \right] \cdot p$$

and it is equivalent to:

$$\begin{aligned} \frac{w}{\beta} + \frac{p}{u'(c)} \cdot \frac{f_G}{M_T^G} \cdot \frac{\left(1 - \frac{\kappa}{q_L}\right)}{\left(1 - \frac{\kappa}{M_{v_L}^L}\right)} \cdot V_N &= pf_G(1 - \sigma) \\ -w(1 - \sigma) + \beta(1 - \sigma) \cdot \left(\frac{w}{\beta} + \frac{p}{u'(c)} \cdot \frac{f_G}{M_T^G} \cdot \frac{\left(1 - \frac{\kappa}{q_L}\right)}{\left(1 - \frac{\kappa}{M_{v_L}^L}\right)} \cdot V_N \right) & \end{aligned}$$

which in turn compared with the steady state version of 68 implies:

$$J_n = \frac{w}{\beta} + \frac{p}{u'(c)} \cdot \frac{f_G}{M_T^G} \cdot \frac{\left(1 - \frac{\kappa}{q_L}\right)}{\left(1 - \frac{\kappa}{M_{v_L}^L}\right)} \cdot V_N.$$

I plug this formula into 67 (in steady state) and get:

$$\beta \mathbb{E}_{a'|a} V_N(Z') \cdot M_{v_L}^L = \exp(a_d) \cdot u'(c)\kappa,$$

which is identical to 81. □

2.3.1 Equilibrium

I define equilibrium in a similar way to [Michaillat and Saez (2015)] (it is the so-called *Fixprice Equilibrium*):⁵⁵

⁵⁴I use the fact that: $\frac{1}{1+\theta_G^{\alpha_G}} = 1 - \frac{M_v^G}{q_G}$.

⁵⁵I follow [Hall (2005)], [Michaillat and Saez (2015)] and I set perfectly sticky prices and wages (so that prices and wages become parameters of the model). This assumption seems to be extremely strong at first glance but on the

Definition 2. A recursive competitive equilibrium (RCE) is price function $p(Z)$, wage function $w(Z)$, value functions $J(n_{-1}, Z)$ and $W(s, Z)$, labor market tightness and product market tightness $\theta_L(Z)$, $\theta_G(Z)$, policy functions $c(s, Z)$, $v(s, Z)$, $s'(s, Z)$, $v_L(n, Z)$ employment choice function $n(n_{-1}, Z)$, discount factor $\Delta(Z', Z)$ and the law of motion $N(Z)$ such that given Markovian processes that govern a_d and a_z :

- 1) Given prices, wages, law of motion and product market tightness $W(s, Z)$ solves the worker's problem and $c(s, Z)$, $v(s, Z)$, $s'(s, Z)$ are the associated policy functions,
- 2) Given prices, wages, law of motion, product market tightness, labor market tightness and discount factor $J(n_{-1}, Z)$ solves the firm's problem and $v_L(n, Z)$, $n(n_{-1}, Z)$ are the associated policy functions,
- 3) Worker's and firm's choices are consistent with aggregate employment, $\theta_L(Z)$ and $\theta_G(Z)$, i.e.:

$$N = n,$$

$$\theta_L(Z) = \frac{1 - (1 - \sigma)N}{v_L(Z)},$$

$$\theta_G(Z) = \frac{\exp(a_z)N - \kappa v_L(Z)}{v(Z)},$$

- 4) Markets clear:

$$s'(s, Z) = 1,$$

$$c(s, Z) = f_G(\theta_G(Z)) \cdot [\exp(a_z)N - \kappa v_L(Z)],$$

- 5) Law of motion for employment holds:

$$N(Z) = (1 - \sigma)N_{-1} + M^L(1 - (1 - \sigma)N_{-1}, v_L(n_{-1}, Z)).$$

- 6) Prices and wages satisfy: 83 and 84.

Equations 63-77, 83 and 84 characterize the competitive equilibrium that has a constrained efficient steady state.

2.4 Optimal allocation with a single friction

In this section I describe the economy with a single friction (i.e., frictional labor market) which is my candidate for the limit of constrained-efficient economies as $\phi \rightarrow 0$ (recall that it is a parameter associated with function $G(v)$). The social planner's problem that corresponds to the model with a

other hand there is no universal theory that would pin down the value of prices (wages) as long as they are elements of bargaining sets when search frictions are in place. Moreover, as it is argued by [Hall (2005)], any fixed values of wages and prices that are elements of bargaining sets can be supported by the concept of Nash equilibrium of the "Demand game". The remaining issue is to choose the exact values for perfectly sticky prices and wages. I think that a natural choice is to set their values at the levels that are consistent with steady state values of prices and wages that decentralize the constrained-efficient allocation in the non-stochastic steady state.

single friction is:

$$V(Z) = \max_{c, v_L, N} \{ \exp(a_d) \cdot u(c) + \beta \mathbb{E}_{a'|a} V(Z') \}$$

subject to :

$$c = \exp(a_z)N - \kappa v_L, \quad (85)$$

$$N = (1 - \sigma)N_{-1} + M^L (1 - (1 - \sigma)N_{-1}, v_L), \quad (86)$$

where V is the value function associated with the planner's problem. I compute the first order condition:

$$\beta \mathbb{E}_{a'|a} V_N(Z') \cdot M_{v_L}^L + \exp(a_d) \cdot u'(c) \cdot (\exp(a_z)M_{v_L}^L - \kappa) = 0. \quad (87)$$

The envelope condition reads:

$$V_N(Z) = \{ \exp(a_d) \cdot u'(c) \cdot \exp(a_z) + \beta \mathbb{E}_{a'|a} V_N(Z') \} (1 - \sigma) [1 - M_U^L]. \quad (88)$$

Equations 85-88 together with 72, 73, 76 and 77 characterize planner's solution.

I am in position to prove the result that is crucial for my decomposition exercise.⁵⁶

Theorem 3. *For $\phi \rightarrow 0$ the allocation corresponding to the constrained efficient solution with two frictions converges to the allocation associated with the optimal outcome with a single friction.*

Proof. I need to show that equations that describe the constrained-efficient outcome with two frictions in the limit when $\phi \rightarrow 0$ are identical to equations that characterize the optimal outcome with a single friction. One can observe that this is true if: $M_T^G \rightarrow 1$, $M_v^G \rightarrow 0$, $f_G \rightarrow 1$ for $\phi \rightarrow 0$.

Observe that if $\phi \rightarrow 0$ then it is optimal for the planner (in the problem with two frictions) to set $v \rightarrow +\infty$ as making visits becomes costless in terms of disutility. This in turn implies that:

$$M_T^G = \frac{v(1 - \frac{T^{\alpha_G}}{T^{\alpha_G} + v^{\alpha_G}})}{(T^{\alpha_G} + v^{\alpha_G})^{\frac{1}{\alpha_G}}} \rightarrow 1, \text{ for } v \rightarrow +\infty.$$

It holds because T is bounded: $0 \leq T \leq 1$ ⁵⁷. Similarly, I have:

$$M_v^G = \frac{T(1 - \frac{v^{\alpha_G}}{T^{\alpha_G} + v^{\alpha_G}})}{(T^{\alpha_G} + v^{\alpha_G})^{\frac{1}{\alpha_G}}} \rightarrow 0, \text{ for } v \rightarrow +\infty,$$

⁵⁶Notice, that the key assumption that is behind this outcome is the functional specification of the matching function M^G .

⁵⁷Observe that if I assumed the Cobb-Douglas specification of the matching function $M^G(v, T) = \gamma_G v^{\eta_G} T^{1-\eta_G}$ (where $\gamma_G > 0$, $0 < \eta_G < 1$) then $M_T^G(v, T) = (1 - \eta_G)\gamma_G (\frac{v}{T})^{\eta_G}$ and hence $M_T^G(v, T) \rightarrow +\infty$ for $v \rightarrow +\infty$ which means that $\lim_{v \rightarrow +\infty} M_T^G \neq 1$. This implies that the limit of economies (when $\phi \rightarrow 0$) does not converge to the candidate for the limit (which is a natural candidate for the case when $\phi = 0$).

The last thing that I need to show is:

$$f_G(\theta_G) = \frac{v}{(T^{\alpha_G} + v^{\alpha_G})^{\frac{1}{\alpha_G}}} \rightarrow 1, \text{ for } v \rightarrow +\infty.$$

which is trivial from what was observed above. This observation completes the proof. \square

Theorem 3 implies that economy described at the beginning of this section is indeed a limit of constrained-efficient economies with two frictions. This fact coupled with Proposition 12 means that the only factors that account for the difference between the unemployment rate in benchmark model and in economy with a single friction are sticky prices/wages and frictions in the product market.

2.4.1 Frictional and Keynesian unemployment

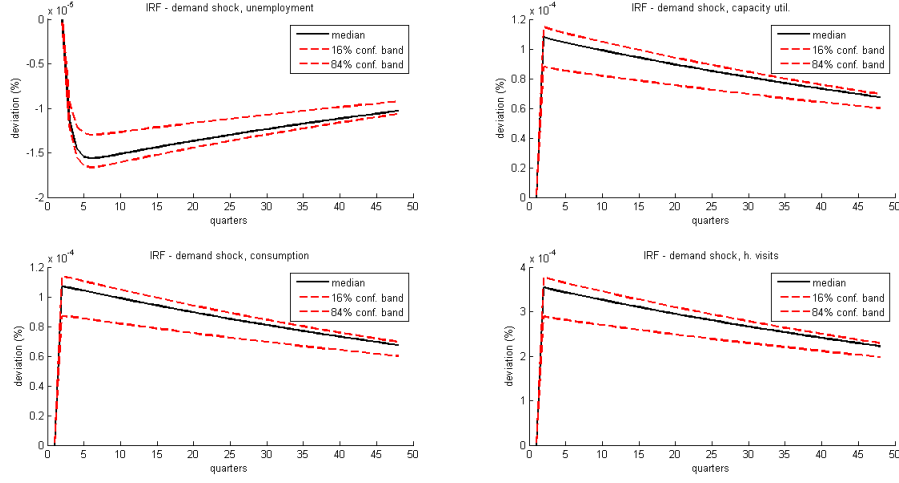
According to Keynesian tradition, the sources of periods characterized by long slumps and high unemployment are: imperfect adjustment of prices, wages and insufficient demand. Since wages do not fall during recessions then demand for labor remains insufficient for the employment level to recover. Symmetrically, these two elements are responsible for amplification of an increase in employment and output during economic booms: prices adjust upwards too slowly which in turn boosts demand. Both features appear in the model of competitive equilibrium with perfectly sticky wages and prices: firstly, price stickiness is introduced by ascribing constant values to prices and wages. Secondly, I have an object (i.e. the number of visits v) that can be interpreted as aggregate demand which may attain low levels if the demand shock a_d decreases.

All this means that if I want to isolate Keynesian "underemployment" or "overemployment" then I need to compare allocation generated by the benchmark model with the model with one friction (in the labor market). This implies the following order of the decomposition exercise. Firstly, I use benchmark model and the Kalman filter to compute the values of shocks that make the model fit the data. Secondly, I use the extracted shocks to simulate the model with a single friction and flexible prices (wages), described at the beginning of Section 2.4 and I obtain the path of unemployment U_f associated with that model. I call it frictional unemployment as its only source are frictions in the labor market.⁵⁸ The difference between the unemployment in model which describes the planner's problem with one friction - U_f and the unemployment rate in the benchmark model - U is called Keynesian overemployment (if the difference is positive) and Keynesian underemployment (if it is equal 0 or negative):

$$\begin{cases} \text{Keynesian overemployment} = U_f - U, & \text{for } U_f > U, \\ \text{Keynesian underemployment} = U_f - U, & \text{for } U_f \leq U. \end{cases}$$

⁵⁸It is easy to show that absent any frictions the unemployment rate is equal to σ .

Figure 13: IRF, demand shock, France



2.5 Calibration and estimation

2.5.1 Missing specifications

I consider the following specification for the utility function u :

$$u(c) = \log(c).$$

2.5.2 Calibration

Calibrated parameters are: σ , β , α_L , α_G , κ , ϕ . The value of σ for Germany, France, Italy and Spain is taken from [Hobijn and Sahin (2007)]. I set the quarterly discount rate $\beta = 0.99$ and $\alpha_L = 1.27$ as [Den Haan et al. (2000)]. I use the steady state version of system 78-82, 72, 73 to find values of four parameters α_G , κ , ϕ for which the moments generated by the model match their empirical equivalents.⁵⁹ In particular, I take: rate of unemployment $1 - (1 - \sigma)N$, capacity utilization of capital $\frac{M^G}{T}$ and labor income share wN_{ss}/pf_GN as criterions for the comparison.^{60 61}

⁵⁹Note, that since I consider steady state version of the competitive allocation with perfectly sticky wages and prices and because I assume that sticky prices and wages are steady state values of prices and wages that decentralize the optimal solution, then stationary allocations 78-82, 72, 73 and 65-75 are identical so I can consider the planner's allocation which is more tractable.

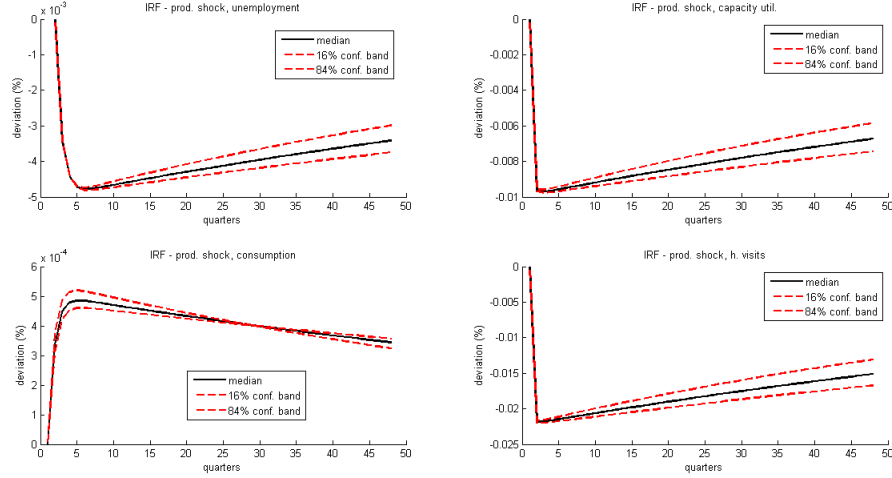
⁶⁰I transform the OECD data on the proportion of unemployed people who remain without a job less than one month to get the quarterly hiring rate $\frac{M_{ss}^L}{U_{ss}}$.

⁶¹Observe that since the production technology is linear in labor then the capacity utilization of capital equals $\frac{M_{ss}^G}{T_{ss}}$.

Table 1: Targeted moments, calibration of α_G , κ , ϕ

	Germany		France		Italy		Spain	
	Data	Model	Data	Model	Data	Model	Data	Model
av. unemployment	9.1%	9.1%	8.9%	9.1%	8.9%	9.1%	15.3%	16.1%
av. cap. utilization	83.9%	83.2%	84.4%	84.7%	74.5%	75.1%	78.1%	78.3%
Labor income share	68%	66%	68%	69%	67%	63%	61%	59%

Figure 14: IRFs, productivity shocks, France



2.5.3 Solution method

I use the method suggested by P. Rendahl to solve the linearized version of the model, i.e. to obtain the following characterization of the dynamical system described by equations 65 to 77:

$$Y_{+1} = \Lambda^* \cdot Y,$$

where Y is a vector of steady state deviations (not necessarily in %) of all variables that appear in 65 to 77 and Λ^* is a transition matrix. The starting point of the algorithm is the linearized version of the model:

$$A \cdot Y_{+1} + B \cdot Y + C Y_{-1} = \mathbb{O}. \quad (89)$$

I take initial guess of transition matrix Λ_0 and after making substitution $Y_{+1} = \Lambda_0 Y$ in 89 I get:

$$Y = -(A\Lambda_0 + B)^{-1} \cdot C \cdot Y_{-1}.$$

Matrix $-(A\Lambda_0 + B)^{-1} \cdot C$ becomes our next candidate for the transition matrix and I denote it by Λ_1 . Then I substitute Λ_1 to 89 and obtain Λ_2 . I repeat this procedure until convergence, i.e. until I find n that satisfies $\max_{i,j} \{|\Lambda_{i,j,n} - \Lambda_{i,j,n-1}|\} < \epsilon$, where ϵ is a small positive number.

2.5.4 Impulse response functions

Figures 13 and 14 present the impulse response functions to a single demand/productivity shock that influences the economy (i.e., benchmark model) in period $t = 0$. Observe that both shocks increase consumption and decrease unemployment. However, they have a different impact when one considers the reaction of the capacity utilization (which is captured by the value of f_G) and the number of visits made by households. Demand shock increases v and (since the adjustment in capacity T is not immediate) it causes an increase in capacity utilization. Productivity shock increases capacity T on impact, boosts the availability of consumption goods and hence households decrease the number of visits that are made (which is costly as it requires search effort captured by disutility $-G(v)$).

2.5.5 Estimation

Bayesian methods are used for estimation of parameters that characterize stochastic processes: ρ_Z , ρ_D and $\Sigma_{2 \times 2}$. It means that I have to estimate four parameter values (as shocks are assumed to be independent).⁶²

Empirical paths of capacity utilization and unemployment are measured signals applied during my estimation. The remaining issue is whether we are able to identify shocks given these two time series. First, observe that impulse responses of unemployment are negative with respect to both shocks. Second, notice that capacity utilization increases when economy is affected by a demand shock and decreases when the system is hit by a productivity shock (Figures 13 and 14). This implies that shocks are orthogonal and hence they can be identified.

2.5.6 Unemployment decomposition

As I have already mentioned, I first use the benchmark model and the Kalman filter to extract the paths of $a_{d,t}$ and $a_{z,t}$ from the data. Second, I use these shocks to simulate the model with a single friction. This gives us the time path of frictional unemployment U_f presented in Figure 15. The difference between these two paths is Keynesian underemployment/overemployment. Notice, that frictional unemployment is procyclical which is intuitive: in periods when the labor market is slack (i.e., recessions) it is easier for firms to find workers and hence the frictional component is relatively low. This result resembles the outcome obtained by [Michaillat (2012)].

In Table 2 one can analyze the structure of unemployment in Germany, France, Italy and Spain. In the first row, I present the steady state values of the total rate of unemployment which is equal to frictional unemployment in my decomposition exercise. In the second row, I analyze the unemployment structure in periods when $U_t > 110\%E(U)$ (which can be thought of as recessions characterized by high unemployment rates).⁶³ It seems that the economy which is the most severely affected by Keynesian unemployment during downturns is Spain: its unemployment structure during economic downturns is different from the one that can be observed in Italy, Germany and France.

⁶²A standard MCMC algorithm is applied to obtain the posterior distributions of estimated parameters. More specific results concerning my estimation can be found in the Appendix. The Kalman Filter is used for computations of the likelihood of empirical data for each iteration of the MCMC procedure.

⁶³To obtain these statistics I simulate the model for 100.000 periods.

Figure 15:

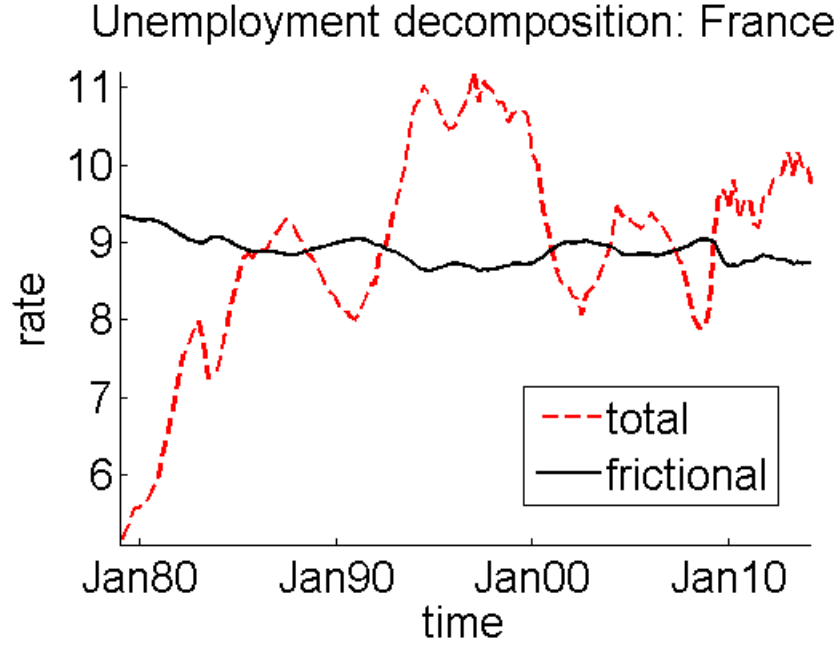


Table 2: Unemployment structure

	Germany	France	Italy	Spain
$\mathbb{E}(U)$	9.1%	9.1%	8.9%	16.1%
$\mathbb{E}\left(\frac{U_f}{U}\right)$ in recession	69.3%	72.8%	71.0%	59.2%

2.6 Concluding remarks

In this paper I have developed a method that allows for the decomposition of unemployment into two components: Keynesian and frictional. Since I conduct the analysis by means of the DSGE model, it is relatively easy to extend this framework to study various issues associated with effects of e.g. fiscal policy or labor market policies on unemployment. My models were used to study the unemployment structure in Germany, France, Italy and Spain. The analysis shows that Keynesian unemployment is a more severe problem during recessions in Spain than in the remaining economies.

My decomposition exercise is significantly different from the one presented in [Michaillat (2012)]. First, I use a modified RBC framework which makes place for the analysis of consumption/saving decision made by households and can be extended to study various labor market institutions. Michaillat used a standard DMP model which abstracts from these aspects of household's behavior and analyzes economy with a single (productivity) shock. Second, I keep the cost of hiring workers κ constant over time - the effective cost of hiring workers is affected solely by endogenous conditions in the labor market captured by q_L (probability of hiring workers). Michaillat assumes that changes in productivity

a_z have a direct effect on the recruitment cost, i.e. it equals $\exp(a_z)\kappa$. This means that in booms $\exp(a_z)\kappa$ increases and hence it is more costly to hire workers. This gives rise to strong procyclical movements in frictional unemployment in his model which are much more moderate in my case.

Appendix

Table 3: Values of calibrated parameters

Parameter	Germany	France	Italy	Spain
β	0.99	0.99	0.99	0.99
σ	0.0315	0.0338	0.0206	0.0597
α_G	2	2	1.4	2
ϕ	0.17	0.15	0.15	0.24
κ	4	3.5	5	2.3

Table 4: Values of estimated parameters: means of prior and posterior distributions

Parameter	Distribution	Ger (prior)	Fr (prior)	It (prior)	Sp (prior)	Ger (post.)	Fr (post.)	It (post.)	Sp (post.)
ρ_D	Beta	0.99	0.99	0.99	0.99	0.982	0.983	0.986	0.975
ρ_Z	Beta	0.99	0.99	0.99	0.99	0.951	0.904	0.991	0.934
σ_D	Inv. gamma	0.01	0.01	0.01	0.01	0.112	0.095	0.102	0.091
σ_Z	Inv. gamma	0.01	0.01	0.01	0.01	0.014	0.011	0.010	0.014

Figure 16:

Unemployment decomposition: Germany

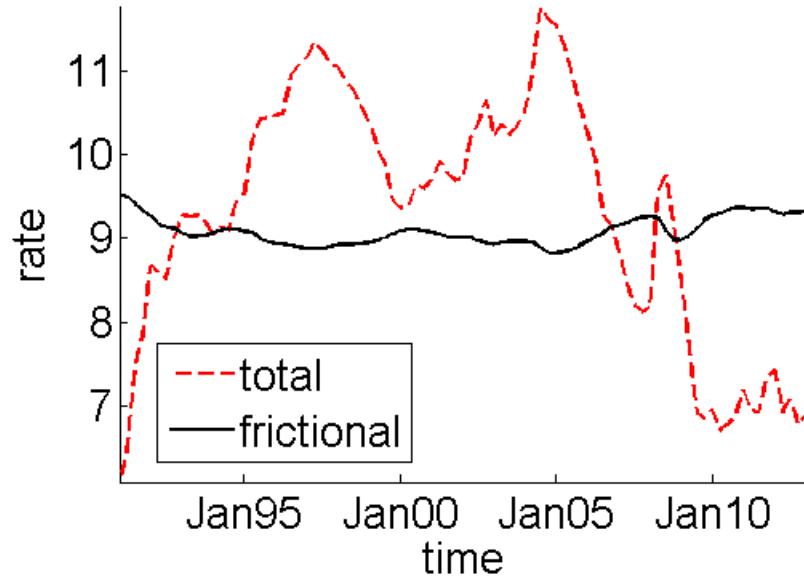


Figure 17:

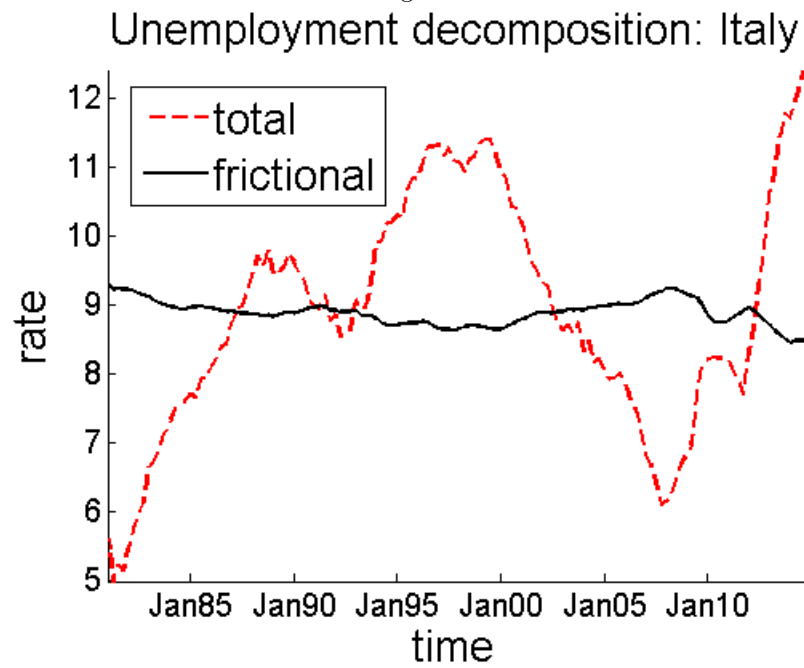
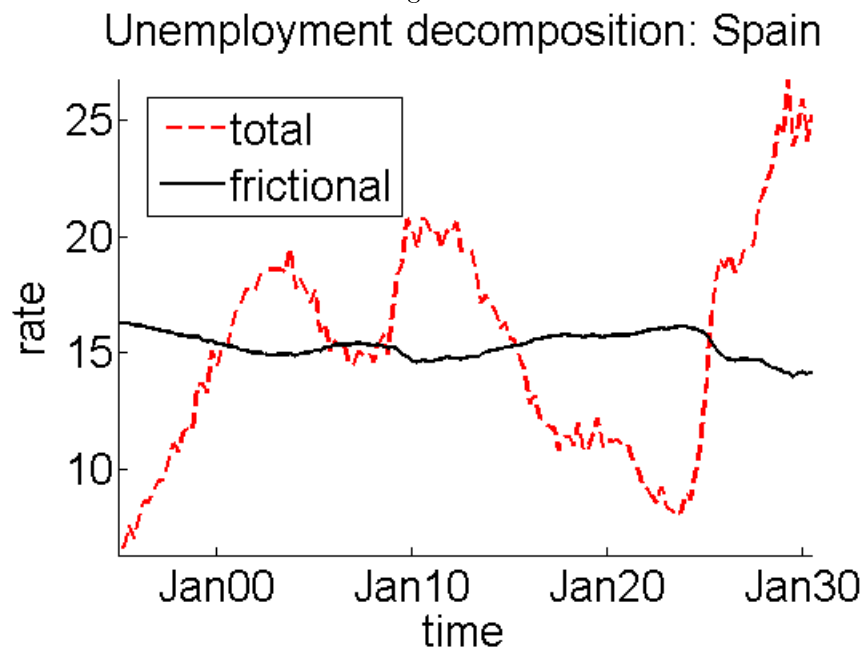


Figure 18:



3 On the Positive Effects of Wasteful Government Expenditures

Abstract

Standard macroeconomic models predict positive values of fiscal multiplier and sharp decreasing relationship between private consumption and government expenditures. The latter result is at odds with empirical evidence. Some recent studies suggest that this negative pattern between private consumption and fiscal purchases is rather moderate or insignificant. More importantly, however, other works indicate that this relationship is positive. I build a tractable, theoretical model that accounts for the qualitative pattern observed in the data: positive government multiplier and increasing relationship between government spending and private consumption. To explain these features I use two ingredients: search frictions in the product market and simple supply chains. The latter element captures the fact that firms need to purchase goods produced by other firms to generate output. It is shown that these two components - in isolation - give rise to the standard prediction found in the theoretical literature: increase in fiscal expenditures crowds out private consumption and increases output. However, the interaction of these elements generates two equilibria and one of them features a positive fiscal multiplier and increasing relationship between government spending and private consumption. This result holds despite the fact that fiscal consumption is assumed to be wasteful and it does not enhance consumers' utility.

3.1 Introduction

I propose a simple framework that uses two ingredients: frictional product market and the presence of supply chains within the firms' sector to study the impact of increase in government consumption on aggregate output and private consumption. It is shown that these two mechanisms - treated separately - imply that increase in fiscal expenditures leads to a drop in private consumption. The interaction of these elements, however, generates two equilibria and one of them features a positive relationship between government spending and private consumption and exhibits a positive fiscal multiplier. It happens due to a novel mechanism: government expansion coordinates firms to scale up their capacities which in turn increases the product market's slackness and decreases the effective price paid by households for consumption goods. This effect is present despite the assumption that government purchases exactly the same goods as those consumed by households and simply wastes them (e.g., by throwing them into ocean).

The first ingredient - frictions in the product market - is used because it gives a precise meaning to the notion of tight markets. This in turn is important when one wants to confront conventional wisdom (i.e., that government consumption crowds out private consumption by increasing market tightness) with outcomes predicted by a theoretical model. Additionally, as discussed by [Bai et al. (2011)], frictional product market gives rise to a situation in which aggregate output is determined not only by the level of production factors but also by demand created by customers (e.g., meals in restaurants

are prepared only if customers show up and order them). This is an intuitive channel through which additional demand generated by government spending may increase output generated by firms.

The second ingredient - presence of supply chains within the firms' sector - accounts for the fact that tighter markets (e.g., as a result of increase in government spending) are not always beneficial for firms. This may seem somewhat counterintuitive because, as discussed above, increased tightness of product markets means that firms find it easier to sell their output. The situation is different, however, if one considers a model in which firms have to search for production factors in frictional product markets: the higher is the market tightness the higher is the effective price at which they purchase production factors. As it is shown later, this channel is essential for the main result of this paper (i.e., the increasing relationship between government spending and private consumption).

The rest of the paper is organized as follows. Section 3.2 presents the literature associated with my analysis. Section 3.3 lays out the model with frictional product market and shows that increase in government spending crowds out private consumption. An analogous result obtains in the model with supply chains that is presented in Section 3.4. Section 3.5 examines the model in which the two ingredients are combined and analyzes expansion in fiscal consumption in this setting. Section 3.6 summarizes the main findings of the paper.

3.2 Literature

Empirical evidence. I do not discuss empirical studies that document positive fiscal multipliers as it seems that there is a broad agreement on this issue. Instead, I concentrate on the strand of literature that describes the relationship between public expenditures and private consumption. An overview of empirical evidence concerning this issue is presented by [Gali et al. (2007)]. They conclude that on the one hand some empirical works find a large, positive and statistically significant response of private consumption to positive changes in fiscal expenditures. On the other hand there are papers that uncover a negative response. The latter effect, however, is generally found to be small and often insignificant. [Blanchard and Perotti (2002)] and [Fatas and Mihov (2001)] use VAR model to study the impact of a persistent rise in government expenditures. Both papers conclude that fiscal expansions cause large increases in private consumption. [Ravn et al. (2012)] use panel structural VAR (applied for four industrialized economies) and document that increase in government consumption raises private consumption. [Fisher and Peters (2010)] identify government spending shocks with statistical innovations to the accumulated excess returns of US military contractors. They document a positive relationship between government spending and private consumption. [Mountford and Uhlig (2004)] find that government expenditures crowd out private investment but their hardly influence consumption. [Ramey and Shapiro (1998)] identify shocks that raise military spending and show that the nondurable consumption displays a small (and barely significant) decline. They find that the consumption of durables exhibits a large increase that is followed by persistent decline. [Ramey (2011)] reexamines the empirical evidence by comparing the two main empirical approaches to estimating the effects of government spending: the VAR approach and the Ramey–Shapiro narrative approach (based on identification of "war dates") and argues that VARs (used by e.g., Blanchard and Perotti) do

not properly measure government spending shocks because changes in government spending are often anticipated long before government spending actually changes.⁶⁴ She presents the evidence that the war dates Granger-cause the VAR shocks but the VAR shocks do not Granger-cause the war dates. This in turn invalidates purely exogenous character of government spending shocks measured by VAR approach and means that results reported by e.g. Blanchard and Perotti not necessarily reflect the pure impact of changes in government spending.

Government expenditures in the RBC model. This strand of literature emphasizes the impact of government consumption on hours worked. This channel plays a key role since in absence of instantaneous adjustment of capital, output can increase (in the short-run) if number of hours worked rises. [Aiyagari et al. (1992)] view jumps in government consumption as exogenous reductions in income. They argue that if the income effect on leisure is zero then changes in government spending has no effect on hours. Moreover, [Aiyagari et al. (1992)] study the impact of both transient and persistent changes in government consumption on labor and find that the contemporaneous effect on hours worked of a persistent rise in government consumption exceeds the impact of the transient one.⁶⁵ This paper, however, does not focus on the effects of government spending on consumption. This issue is discussed in an important work of [Baxter and King (1993)]. They investigate the impact of permanent and temporary expansions in government spending and find that the former can lead to output multipliers (both short-run and long-run) that exceed one. As in [Aiyagari et al. (1992)], [Baxter and King (1993)] highlight the role of increase in hours worked that gives rise to the multiplier mechanism. Additionally, they notice that a rise in hours that follows a permanent fiscal expansion increases the marginal productivity of capital. This in turn gives incentives to accumulate capital which in turn boosts private investment. This effect coupled with the standard effect of absorption of resources by the government leads to lower private consumption. This drop is particularly severe right after the change in fiscal expenditures and it dampens as economy converges to the new steady state. This happens because higher capital stock and increased number of hours worked generate greater amount of resources in economy each period. I show that an increase in private consumption following a fiscal expansion is possible in a model with search frictions in the market for products and simple supply structure. Moreover, the associated rise in output occurs in absence of the dynamic hours worked - capital interactions.

Government expenditures in the New Keynesian (NK) model. The fact that standard DSGE models predicted a decreasing relationship between private consumption and expansions in

⁶⁴“War dates” are episodes where Business Week suddenly began to forecast large rises in defense spending induced by major political events that were unrelated to the state of the U.S. economy.

⁶⁵This happens because agents faced by a transient increase in government consumption cut their investment expenditures by more than in the situation when the increase is permanent. They decrease investment because jump in government consumption decreases the amount of resources available in economy and hence they do it for the consumption-smoothing motives. The decline in investment is lower in case of persistent changes in government spending as agents expect that the amount of resources will remain lower (due to persistent fiscal expansion) in the next period so they decide not to cut investment by so much “today” as it leads to decrease in aggregate capacity “tomorrow” and to a further shrinkage of the resource constraint. This in turn implies that the amount of resources available for consumption today is lower in case of persistent change in government expenditures which in turn means that households decide to work more today than in case of a transient fiscal expansion. This means that increase in hours worked is greater when the growth in fiscal spending is more persistent.

fiscal spending (which was at odds with empirical evidence) became a motivation for the paper of [Gali et al. (2007)]. They study an extended version of the standard NK model. In particular, they allow for the presence of rule-of-thumb consumers that spend their entire labor income on consumption. This assumption implies that expansion in government purchases is able to raise aggregate consumption through the induced increase in employment and the rise in real wages. This is because the latter two factors boost labor income and hence they raise consumption of hand-to-mouth consumers. This in turn boosts aggregate demand, output, employment and wages even further so that multiplier effects emerge.

Government expenditures and the Zero Lower Bound (ZLB). This literature analyzes the impact of the government spending in the situation when the short-term nominal interest rate is zero and the economy experiences excess deflation. This leads to higher real interest rates and makes households postpone their consumption spending. Output becomes demand determined.

The first channel through which various policies affect the economy that finds itself at the ZLB is the expected inflation channel. The idea (see, e.g., [Eggertsson (2010)]) is that policies that aim at boosting aggregate supply are counterproductive as they reinforce deflationary expectations and hence they increase real interest rates even further. The effects of policies that rise the aggregate demand (e.g., government expenditures) are just the opposite. Eggertsson uses a standard New Keynesian model to show that a temporary increase by one dollar in fiscal spending directed at goods that are imperfect substitutes with private consumption leads to output growth by 2.3 dollars. The key driving force of this effect is that expectations about future policy (government commits to sustain spending until the recession characterized by the ZLB is over) in all future states in which the ZLB binds inflates the price level in those periods. This in turn creates inflationary expectations in the current period and causes a drop in the real interest rates which stimulates aggregate demand. Notice that in the NK model without capital increase in output is splitted solely between private and public consumption. This means that if the multiplier is higher than one then consumption increases when government consumption rises. Increase in private consumption that follows fiscal expansion that is presented in my analysis does not require the assumption about the ZLB. It is worth mentioning that Eggertsson analysis implies that negative supply shocks are expansionary at the ZLB. This prediction was tested by [Wieland (2016)]. He used the episodes of the Great East Japan earthquake and global oil supply shocks that occurred in the ZLB environment to show that Eggertsson's results are not consistent with empirical observations. Additionally, as shown by [Bachmann et al. (2015)], US households' readiness to spend more in response to changes in inflation expectations is statistically insignificant inside a liquidity trap. This implies that the empirical support for the expected inflation channel used by Eggertsson in his theoretical analysis is not very strong.

The second channel described in the context of the ZLB is associated with equilibrium unemployment dynamics and was described by [Rendahl (2015)]. The mechanism that is present in his model is based on two ingredients. First, he exploits the fact that when short-term interest rates are zero then output is largely determined by demand. The second ingredient is frictional labor market. The interplay between those two components and the increase in government spending works as follows: since at the ZLB aggregate product is determined by demand then increase in government spending

raises output and decreases unemployment rate in the present. Because of frictions in the labor market, the decrease in unemployment is persistent and thus future unemployment rates fall, too. This in turn means, that agents' income increases in the future. Since they exhibit consumption-smoothing behavior then they a rise in future income feeds back to an increase in present consumption. This boosts aggregate demand even further and decreases current and future unemployment rates even further. Rendahl uses an extended version of the standard Diamond-Mortensen-Pissarides model to calculate the fiscal multiplier associated with mechanism described above and reports that its value is slightly below 1.9. Moreover, similarly to my analysis, he finds that in case of a prolonged liquidity trap, hike in government expenditures boosts private consumption. What is different in my model is that the increase in private consumption in response to jump in fiscal spending does not rely on any dynamic interactions and the assumption about the ZLB.

Models with search frictions in the product market. One of the key ingredients in my analysis is the frictional product market. This environment was studied by [Bai et al. (2011)] and [Michaillat and Saez (2015)]. [Bai et al. (2011)] show that demand shocks are responsible for the TFP volatility if the product market frictions are in place. [Michaillat and Saez (2015)] develop a theoretical, continuous-time model with search frictions both in market for goods and in labor market, use it to conduct a comparative-statics analysis and study the sources of labor market fluctuations in the US.

Models with multiple equilibria. My work is also related to articles that describe models with multiple equilibria. I propose a novel source of multiplicity that arises from the interaction between search frictions on the product market and the fact that firms need to visit their suppliers and thus they are subject to those frictions, too. In a large class of models ([Benhabib and Farmer (1994)], [Farmer and Guo (1994)], [Diamond (1982)], [Diamond and Fudenberg (1989)]), multiplicity obtains because of increasing returns to scale either in production or in matching. These features are absent in my analysis. In what follows I concentrate on two papers that study the impact of fiscal spending in environments that exhibit multiple equilibria.

In a seminal paper, [Diamond (1982)] proposes a model with search frictions that is subject to thick market externality. This means that returns to participating in the market are higher when the number of agents in the market increases.⁶⁶ If an agent sees that the number of potential trading partners is higher then the return on his output grows as search frictions in the market are lower. Therefore he chooses a higher cutoff for the cost of production opportunities drawn from a certain distribution. This increases his output (on average) and means that he enters the market more frequently. This feedback loop gives rise to multiple equilibria. He finds that government intervention that leads to higher cutoff value of the cost of production opportunities improves welfare in all steady state equilibria. This is because there is only the thick market externality in Diamond's framework. This is the reason for which policy recommendations are relatively straightforward. In my model, there is a role for congestion effect in addition to thick market externality: on the one hand, increase in market tightness rises the probability that firms find customers which increase the returns from output (thick market externality); on the other hand, however, they find it harder to get resources needed to generate output

⁶⁶This mechanism hinges on the assumption made by Diamond that agents cannot consume their own output and they need to find trading partners in the market to exchange their product.

(congestion effect). This gives rise to situations in which interventions that result in increased market tightness are not always desired. This happens also because higher tightness is always harmful for households in my model.

[Schaal and Taschereau-Dumouchel (2015)] study the interaction between demand externalities and non-convexities in production decisions that give rise to multiple equilibria in an otherwise standard RBC model. They find that once government spending have an impact on labor supply decisions of households then they may result in welfare gains. The first part of their story is familiar from the standard RBC model: households decide to increase labor supply in response to government spending (that decreases their income). This in turn puts a downward pressure on wages and hence firms are more tempted to use high capacity (non-convex decision) which alleviates the coordination problem (firms are more likely to choose higher capacity level). They do not discuss the impact of government expenditures on private consumption, though.

3.3 Model with frictional product market

In this section I present a tractable static model with frictional product market and study the impact of changes in government spending in this setting. It is based on framework presented by [Michaillat and Saez (2015)].

General setting. Economy is populated by a continuum of households and firms of measure one each. There are two types of goods traded in economy: a non-produced good (which is a numeraire) and a good that is manufactured by firms. Each firm has capacity normalized to 1 and it is able to generate output without costs. The non-produced good is traded on a perfectly competitive market, whereas the market on which the produced good is traded is characterized by search frictions (specified later).

Households. Households derive utility from consumption of both types of goods. In particular, their preferences are specified as follows:

$$u(c, m) = \log c + \chi \log m \tag{90}$$

where c denotes the consumed amount of manufactured goods and m denotes the number of units of non-produced goods that are consumed. Logarithmic specification of the utility function is assumed to simplify calculations. Search frictions are modeled as in [Michaillat and Saez (2015)]: to purchase produced goods, household has to visit firms - each visit costs $\phi > 0$ units of the manufactured goods and number of visits made by a household is v .⁶⁷ Due to presence of search frictions some visits are successful and some are not. If a visit is successful then the number of manufactured goods purchased by household is one and it occurs with probability $q(x)$ where x is tightness in the market for products (defined later) and it is taken by households as given. This means that the following relationship

⁶⁷An alternative way of specifying search costs (i.e., in terms of disutility from search activities) is described in [Bai et al. (2011)]. As I show in the Appendix, main results from the core text hold under their specification of search costs, too.

between the number of visits and consumed goods holds:

$$c + \phi v = q(x)v. \quad (91)$$

I abstract from randomness at the individual level throughout the paper which means that all households get exactly $q(x)v$ of produced goods (this assumption applies to firms, too). Let us define the wedge in the market for manufactured goods as $\tau(x) = \frac{\phi}{q(x)-\phi}$. Household's income consists of two components: endowment μ of the non-produced good and profits Π generated by firm(s) and it is spend on m , c and to cover the costs associated with visits. This means that the budget constraint reads:

$$pc + p\phi v + m = \mu + \Pi \quad (92)$$

where p is price of produced goods. By substituting 91 into 92 and using the definition of $\tau(x)$ we get:

$$p(1 + \tau(x))c + m = \mu + \Pi. \quad (93)$$

Household maximizes 90 subject to 93 with respect to c and m . This, together with the market clearing condition for the non-produced good (i.e., $m = \mu$), yields the following formula for the optimal choice of c :

$$c = \frac{\mu}{\chi p(1 + \tau(x))}. \quad (94)$$

Firms. In this simple model firms have capacity normalized to 1. This means that they would like to produce and sell one unit of produced goods. Since there are search frictions in place, they are able to sell a proportion $f(x)$ of their products. It is assumed that unsold goods are wasted. This means that firm's profit is:

$$\Pi = p \cdot f(x) \cdot 1.$$

Search frictions and price-setting mechanism. The aggregate number of successful trades on the product market is given by: $M(1, v)$, where M is increasing in both arguments, it is strictly concave and it exhibits constant returns to scale. This means that firms finds a customer with probability given by:

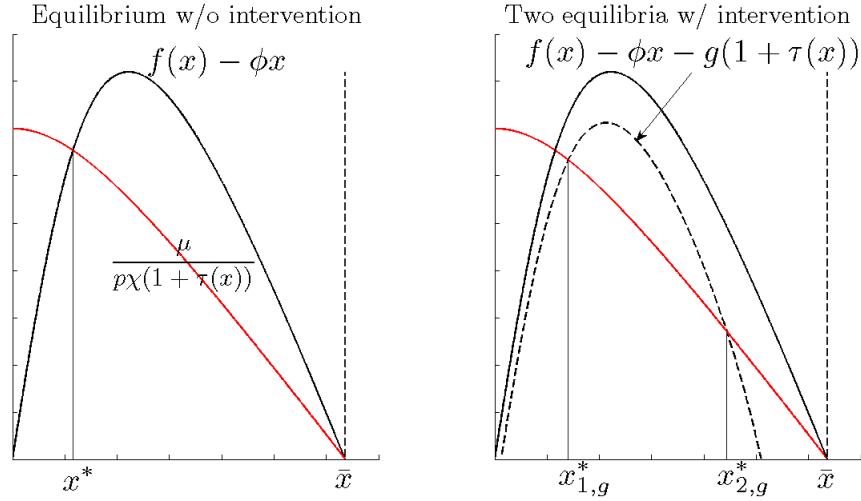
$$f(x) = \frac{M(1, v)}{1} = M\left(1, \frac{v}{1}\right) = M(1, x).$$

since the tightness of the product market is defined as $x = \frac{v}{1}$. Probability that household's visit is successful reads:

$$q(x) = \frac{M(1, v)}{v} = M\left(\frac{1}{v}, 1\right) = M\left(\frac{1}{x}, 1\right).$$

Since there is no universal theory that pins down the price in the situation when the trade is decentralized, I assume that prices are perfectly rigid, i.e. p enters into the model as a strictly positive parameter. This assumption is made for simplicity but the main result of the paper (i.e., private consumption can increase with government expenditures) holds under more general conditions concerning the price-setting mechanism, too.

Figure 19: Model with frictional product market



Equilibrium. The resource constraint for the produced good is:

$$c + \phi v = f(x) \cdot 1. \quad (95)$$

Using the definition of tightness in the product market and combining it with 94 and 95 yields:

$$\frac{\mu}{\chi p (1 + \tau(x))} = f(x) - \phi x. \quad (96)$$

Equation 96 characterizes the equilibrium value of x . Since 94 can be used to reformulate 95 to obtain:⁶⁸

$$\frac{\mu}{p\chi} = f(x)$$

By assuming that $f(\bar{x}) > \frac{\mu}{p\chi}$ (where \bar{x} solves $q(\bar{x}) = \phi$) and by observing that $f(0) = 0$ and $f' > 0$ it is clear that solution $x^* \in (0, \bar{x})$ to 96 exists and is unique. First panel of Figure 19 illustrates equilibrium condition 96.⁶⁹

Effects of an increase in fiscal spending. Let us analyze the impact of increase in government spending from 0 to some positive number $g > 0$ that is financed by lump-sum taxes levied on households. I assume that government consumption is financed by lump-sum taxes - it seems that it is a natural benchmark for isolating the theoretical effects of rise of government spending on aggregate

⁶⁸The result that the total amount of goods purchased by households (i.e., $c(1 + \tau(x))$) is constant (i.e., equal to $\frac{\mu}{p\chi}$) follows by the log specification of preferences.

⁶⁹I have used the following parameter values to prepare the plots in this section: $\phi = 0.3$, $\mu = 1$, $\chi = 1$, $p = 2$, $L = 2$ (parameter associated with the Den Haan - Ramey - Watson specification of the matching function), $\alpha = 0.5$, $g = 0.03$.

activity. It is assumed that government buys produced goods and they are thrown into ocean. Symmetrically to households, it is assumed that government has to visit firms on the decentralized and frictional market to purchase goods. This means that if government wants to buy g of goods it has to make v_G visits where v_G satisfies:

$$g + \phi v_G = q(x) \cdot v_G.$$

We have to modify the definition of tightness x :

$$x = \frac{v + v_G}{1}.$$

Using the definition of $\tau(x)$ enables us to reformulate expression for the "gross" fiscal expenditures:

$$g + \phi v_G = g \cdot (1 + \tau(x)) \equiv G(x).$$

Household's budget constraint is:

$$p(1 + \tau(x))c + m + T = \mu + \Pi$$

where $T = p \cdot G(x)$ guarantees that government runs a balanced budget. The resource constraint for economy with $g > 0$ becomes:

$$c + \phi v + g + \phi v_G = f(x) \cdot 1.$$

This combined with the optimal policy of households yields:

$$\frac{\mu}{\chi p(1 + \tau(x))} = f(x) - \phi x - G(x). \quad (97)$$

Second panel of Figure 19 illustrates equation 97. This equilibrium condition can be reformulated to get:

$$\frac{\mu}{p\chi} + G(x) = f(x). \quad (98)$$

Observe that since: $G(x)$ is an increasing function on $[0, \bar{x})$, $\lim_{x \rightarrow \bar{x}} G(x) = +\infty$, $f(\bar{x}) > \frac{\mu}{p\chi}$ holds and by assuming that g is sufficiently small means that equation 98 has two solutions. I denote them by $x_{1,g}^*$ and $x_{2,g}^*$. Without loss of generality I consider the situation when $x_{1,g}^* < x_{2,g}^*$. In what follows I ignore the equilibrium characterized by $x_{2,g}^*$. It is because response of the economy to increase in g is "discontinuous" - an arbitrarily small value $g > 0$ leads to significant change from x^* to $x_{2,g}^*$. Additionally, comparison of x^* and $x_{2,g}^*$ excludes the possibility of using comparative statics that is based on calculus and smoothness of functions.

Let us concentrate on the relationship between x^* and $x_{1,g}^*$, then. A simple application of the Implicit Function Theorem for equation 98 in the neighborhood of $x^* = x_{1,g=0}^*$ implies that:

$$\frac{dx_{1,g}^*}{dg} > 0$$

which means that government intervention increases tightness on the product market. On the one hand, a rise in tightness leads to growth in output since $f(x) \cdot 1$ is an increasing function. The intuition behind this outcome is straightforward: government spending boosts the demand for manufactured goods and hence it increases the rate/probability at which firms sell their output. Since the capacity of firms is fixed then aggregate output rises. On the other hand, however, since $\tau(x)$ grows in x then fiscal expansion causes a drop in private consumption (by equation 94). This occurs even despite the increase in $f(x) \cdot 1$ - the amount of goods available in economy. It happens partly because of the strong assumption that government buys exactly the same type of goods (and throws them into ocean) as those consumed by households. The absorption of resources that could have been used by the private sector decreases the rate at which consumers purchase goods (i.e., $q(x)$ falls) and raises the effective price of manufactured goods: $p(1 + \tau(x))$.

One comment is in order here. Observe that if we change the assumption that the initial amount of fiscal spending is zero and replace it with a positive value then the model with a single friction exhibits two equilibria (see Figure 19). In equilibrium associated with higher tightness (i.e., $x_{2,g}^*$), further increases in g (government consumption) cause drops in tightness which rises private consumption (see formula 94). One could argue that this fact indicates that the model with a single ingredient (search friction on the product market) is able to reproduce the pattern observed in the data and hence the addition of second element (supply networks) is redundant. To see why it is not the case observe that the equivalent of aggregate output in this model is $f(x) \cdot 1$ (with $f' > 0$). As it has been discussed, $x_{2,g}^*$ drops when g rises which in turn implies a decrease in $f(x)$ and hence causes a decline in output. So the economy with a single ingredient (search frictions on the product market) is not able to reproduce the pattern we want to obtain because it generates either negative fiscal multiplier and positive response in private consumption or just the opposite pair of effects.

3.4 Model with supply networks

In this section I describe a model in which each firm needs to purchase goods from other firms to get resources needed to generate output. Contrary to the model presented in the preceding section, environment developed in this part is characterized by the frictionless product market and flexible prices.

General setting. Types of agents, types of goods and sizes of populations remain unchanged in comparison to the model presented in previous section. There are two important differences though. First, both markets are perfectly competitive. Second, firms' production technology becomes more complicated. In particular, they are not simple "Lucas trees" anymore. To generate output they need to buy goods manufactured by other firms.

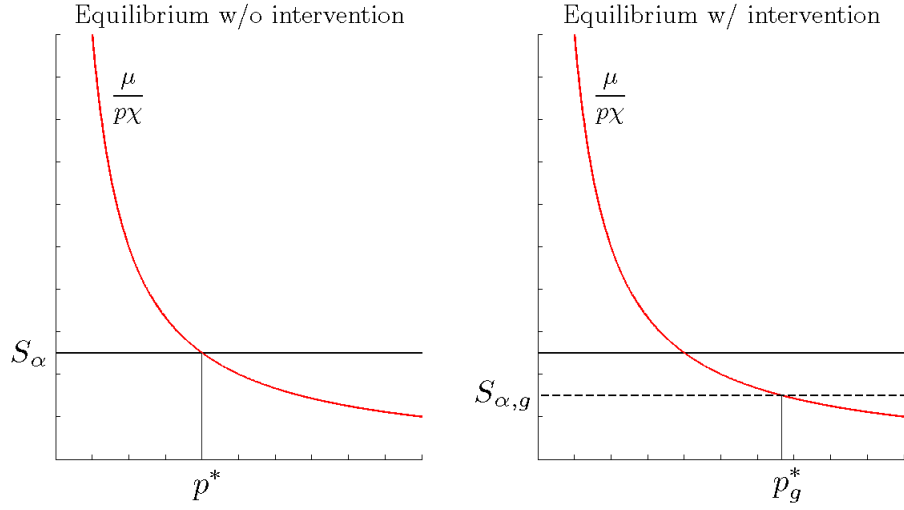
Households. Households' preferences are the same:

$$u(c, m) = \log c + \chi \log m \tag{99}$$

The budget constraint is:

$$pc + m = \mu + \Pi \tag{100}$$

Figure 20: Model with simple supply chains



Household maximizes 99 subject to 100 with respect to c and m . This, together with the market clearing condition for the non-produced good (i.e., $m = \mu$), yields the following formula for the optimal choice of c :

$$c = \frac{\mu}{\chi p}. \quad (101)$$

Firms. Firms operate the concave technology (described by parameter $\alpha \in (0, 1)$) that transforms y goods purchased from other firms into y^α of their own product. Profit function is:

$$\Pi = \max_y p \cdot (y^\alpha - y)$$

and the associated FOC implies that:

$$y_{opt} = \alpha^{\frac{1}{1-\alpha}}.$$

Equilibrium. Resource constraint for manufactured goods is:

$$c = y_{opt}^\alpha - y_{opt}. \quad (102)$$

The RHS of 102 is the amount of final goods available for households. It accounts for value added created by all firms in the economy and hence it is an analog to the standard notion of GDP. Plugging optimal polices into the model yields the following formula for p that characterizes the equilibrium in this simple economy:

$$\frac{\mu}{\chi p} = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}.$$

First panel of Figure 20 presents a graphical illustration of this equation (I define $S_\alpha = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}$).⁷⁰ Since $\alpha \in (0, 1)$ then the RHS of the formula above is positive and there exists a unique price p^* that solves it.

Effects of an increase in fiscal spending. Let us analyze the impact of increase in government spending from 0 to some positive number $g > 0$ that is financed by lump-sum taxes levied on households. If we modify the resource constraint for the produced good we get:

$$c + g = y_{opt}^\alpha - y_{opt}.$$

This equation combined with optimal plans of agents yields:

$$\frac{\mu}{\chi p} = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} - g. \quad (103)$$

The RHS of 103 is denoted by $S_{\alpha,g}$ and the impact of fiscal intervention is presented in Figure 20 (right panel). It is self-evident that since aggregate supply $y_{opt}^\alpha - y_{opt}$ remains unaffected by changes in g then increase in fiscal spending reduces the amount of goods available for households. It is a pure crowding-out process and it is driven by the effect of increase in g on prices - it leads to increase in price ($p^* < p_g^*$) and by equation 101 private consumption drops.

3.5 Model with frictional product market and supply networks

In this section I study the interplay between two features that have been investigated separately so far. As a result I obtain a model with two equilibria and each of them exhibits a different reaction to hikes in fiscal expenditures.

General setting. Types of agents, types of goods and sizes of populations remain unchanged in comparison to models presented in previous sections. In this part, however, I combine two elements that were studied separately before: I assume that there are search frictions in the product market and that firms need to search for their suppliers.

Households. Consumers purchase non-produced goods on a perfectly competitive market and manufactured good on the frictional market. Household's problem is the same as in Section 3.3. This means that household's behavior can be summarized by the following FOC (see equation 94):

$$c = \frac{\mu}{\chi p (1 + \tau(x))}. \quad (104)$$

Firms. In this section I assume that firms are visited not only by households that want to get manufactured goods but part of their output is sold to other firms that search for resources needed to generate their own products. This means that $f(x)$ is probability that a firm is visited by consumers or firms. Their production technology is described by the concave function that transforms y units of "raw materials" purchased from other manufacturers into y^α units of goods. Symmetrically to households,

⁷⁰I have used the following parameter values to prepare the plots in this section: $\mu = 0.02$, $\chi = 1$, $p = 2$, $\alpha = 0.5$, $g = 0.1$.

firms have to incur costs of visits v_y when they seek for their inputs. This means that firm's problem can be formalized as follows:

$$\begin{aligned} & \max_{y, v_y} (pf(x)y^\alpha - py - p\phi v_y) \\ & \text{subject to :} \\ & y + \phi v_y = q(x)v_y. \end{aligned} \tag{105}$$

I use constraint 105 and the definition of $\tau(x)$ to eliminate v_y from the maximization problem:

$$\max_y (pf(x)y^\alpha - p(1 + \tau(x))y). \tag{106}$$

Observe that increase in tightness x has two opposite effects on the situation of firms. On the one hand, higher x means that firms sell their output more easily. On the other hand, however, increase in x means that the effective price of inputs $p(1 + \tau(x))$ increases. Optimal solution y^* combines those two effects:

$$y^* = \left[\frac{\alpha f(x)}{1 + \tau(x)} \right]^{\frac{1}{1-\alpha}}. \tag{107}$$

Equation 107 describes firm's demand for inputs.

Search frictions and price-setting mechanism. Search frictions are almost the same as in Section 3.3. The only difference is the definition of tightness which becomes:

$$x = \frac{v + v_y}{y^\alpha}.$$

Price p is fixed and it is a positive parameter.

Equilibrium. Observe that the aggregate amount of resources available for households (final goods used by consumers for consumption and for covering search costs) is:

$$\begin{aligned} & f(x)(y^*)^\alpha - (1 + \tau(x))y^* \\ &= \frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} \right) (f(x))^{\frac{1}{1-\alpha}}}{(1 + \tau(x))^{\frac{\alpha}{1-\alpha}}} \equiv Y(x). \end{aligned}$$

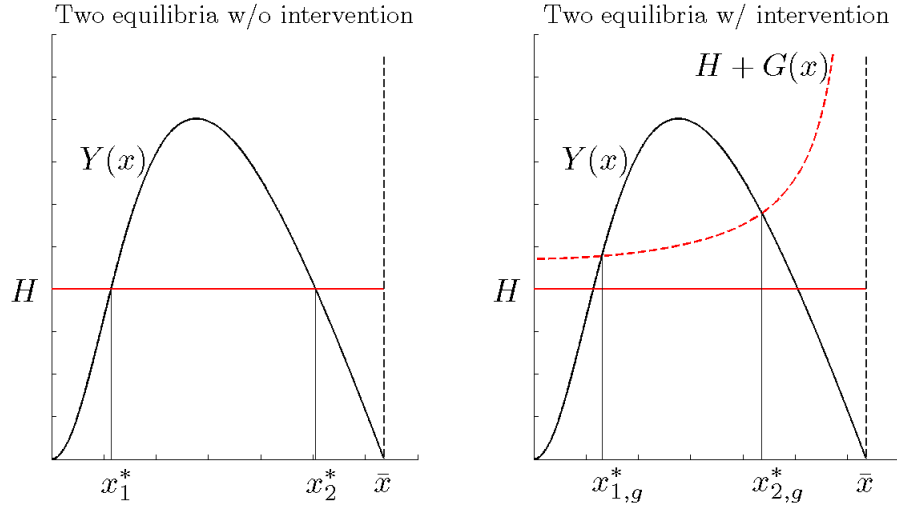
In the Appendix I show that $Y'(x) > 0$ for $x \in [0, x_P)$ and $Y'(x) \leq 0$ for $x \in [x_P, \bar{x}]$ where $0 < x_P < \bar{x}$ and \bar{x} solves $q(\bar{x}) = \phi$. Moreover, it is easy to see that: $Y(x) > 0$ (for $x \in (0, \bar{x})$), $Y(0) = 0$ and $\lim_{x \rightarrow \bar{x}} Y(x) = 0$ (the latter follows by the definition of wedge $\tau(x)$). From equation 104 we get:

$$c(1 + \tau(x)) = \frac{\mu}{\chi p}.$$

Market clearing condition for manufactured goods is thus:

$$\frac{\mu}{\chi p} = Y(x). \tag{108}$$

Figure 21: Static model with frictions and supply chains



From what was said about function $Y(x)$ it is clear that 108 has two solutions provided that μ is sufficiently low (or, alternatively, χ is high enough). Let us denote them by x_1^* and x_2^* (without loss of generality $x_1^* < x_2^*$). To economize on notation I denote $H \equiv \frac{\mu}{\chi p}$. Observe that aggregate output of final goods is equal in both equilibria. Condition 108 is presented in the left panel of Figure 21.⁷¹

Effects of an increase in fiscal spending. In this part analyze the impact of an increase in government expenditures on allocations associated with equilibria characterized by x_1^* and x_2^* . Similarly to the case analyzed in Section 3.3, government sets number $g > 0$ (amount of goods that are thrown into ocean) and hence it has to purchase

$$G(x) = (1 + \tau(x))g$$

of manufactured goods and makes $v_G = \frac{g}{q(x) - \phi}$ visits. The resource constraint for this type of goods is:

$$\frac{\mu}{\chi p} + G(x) = Y(x) \quad (109)$$

and market tightness is redefined in the following way:

$$x = \frac{v + v_y + v_g}{y^\alpha}$$

Since $G'(x) > 0$ then it is easy to see that (as long as g is sufficiently small) the property that the model has multiple equilibria is preserved. They are characterized by numbers $x_{1,g}^*$ and $x_{2,g}^*$ (without loss of

⁷¹I have used the following parameter values to prepare the plots in this section: $\phi = 0.3$, $\mu = 1$, $\chi = 1$, $p = 25$, $L = 2$ (parameter associated with the Den Haan - Ramey - Watson specification of the matching function), $\alpha = 0.5$, $g = 0.005$.

generality $x_{1,g}^* < x_{2,g}^*$). Equation 109 is shown in the right panel of Figure 21. Same as before, I ignore the possibility that agents' expectations switch so that economy behaves in a non-continuous manner after the intervention. For instance, I exclude the possibility that economy that is characterized by x_1^* when $g = 0$ exhibits value $x_{2,g}^*$ of product market tightness for $g > 0$.

First, observe that both equilibria exhibit positive fiscal multipliers as: $Y(x_1^*) < Y(x_{1,g}^*)$ and $Y(x_2^*) < Y(x_{2,g}^*)$. There is, however, an important qualitative difference between their reaction to increase in g . Notice that $x_{1,g}^* > x_1^*$ - i.e., market tightness increases in g . This resembles the effects of fiscal expansion analyzed in Section 3.3: government consumption g reduces market slackness and hence both firms and households find it harder to purchase manufactured goods as their effective price $p(1 + \tau(x))$ grows. By equation 104 it can be concluded that private consumption drops.

Let me concentrate on a more interesting case that pertains to equilibrium characterized by x_2^* . As I have already mentioned, aggregate output of final goods grows (in this equilibrium) in response to rise in government consumption g . More importantly and somewhat counterintuitively, an increase in g causes a fall in tightness: $x_{2,g}^* < x_2^*$ (see Figure 21). To understand why it happens let us analyze equation 107. In particular, observe that firm's demand for inputs can be reformulated in the following way:

$$y^*(x) = \left[\frac{\alpha f(x)}{1 + \tau(x)} \right]^{\frac{1}{1-\alpha}} = \frac{Y(x)}{1 + \tau(x)} \frac{\alpha^{\frac{1}{1-\alpha}}}{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} \right)}. \quad (110)$$

Recall, that in equilibrium described by x_2^* , fiscal intervention increases output of final goods $Y(x)$ and decreases $\tau(x)$. These two forces work in the same direction and hence $y^*(x)$ grows.⁷² It means that in equilibrium characterized by x_2^* the reaction of output and capacity to change in g is so strong that tightness (given by $x = \frac{v+v_u+v_g}{y^\alpha}$) falls despite the fact that v_g increases. The drop in $\tau(x)$ compensates the decrease in $f(x)$ (the probability that firms sell their output successfully) and firms decide to expand their output by scaling up their capacity $y^*(x)$. This in turn decreases the effective price $p(1 + \tau(x))$ faced by other firms that choose to increase their capacity, too. In short, an increase in government expenditures coordinates firms to raise their capacities.

Let us take a closer look at technical aspects that are behind the mechanism described above. First, let us rewrite the equation that describes firm's profits for some level of y :

$$pf(x)y^\alpha - p(1 + \tau(x))y.$$

First, observe that f is concave. It is an immediate consequence of the assumption about concavity of M . Second, notice that τ is convex. This fact requires more subtle argument which is provided in the Appendix. These properties imply that in equilibrium characterized by x_2^* (i.e., when tightness is relatively high) a downward change in x causes a small drop in $f(x)$ which is compensated by a large drop in $p(1 + \tau(x))$. This makes firms expand their capacities by increasing their level of y .

⁷²Notice that change in $y^*(x)$ in case of equilibrium described by $x_{1,g}^*$ is ambiguous as both the numerator and the denominator of 110 increase.

3.6 Conclusions

I have presented a simple framework in which expansion in wasteful government expenditures can lead to an increase in private consumption and a positive fiscal multiplier. To obtain this outcome I have used two simple building blocks - search frictions on the product market characterized by the matching technology that exhibits constant returns to scale and simple supply networks - to capture the fact that firms generate output by using the resources produced by other enterprises. The result emerges because government intervention coordinates firms to increase their capacities. This in turn relaxes their "search constraint" that appear in the model as firms need to search for resources produced by other firms. It is because a firm that faces a decrease in tightness on markets on which it buys resources needed for its production activities (caused by a rise in capacities of its suppliers) incurs lower production costs. This in turn creates incentives to scale up its own capacity despite the fact that the probability at which To obtain this pattern of consumption response I have used two ingredients: search frictions in the product market and the presence of simple supply chains structure among firms. Neither of them is able to induce the positive relationship between private consumption and government spending if it is isolated from the other one.

Appendix

Properties of function $Y(x)$. Let's calculate the derivative of $Y(x)$:

$$Y'(x) = \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} \right) \frac{\frac{1}{1-\alpha} f(x)^{\frac{1}{1-\alpha}-1} f'(x) (1 + \tau(x))^{\frac{\alpha}{1-\alpha}} - \frac{\alpha}{1-\alpha} f(x)^{\frac{1}{1-\alpha}} \tau'(x) (1 + \tau(x))^{\frac{\alpha}{1-\alpha}-1}}{(1 + \tau(x))^{\frac{2\alpha}{1-\alpha}}}.$$

We have to concentrate on the sign of expression:

$$\frac{f'(x)}{f(x)} - \alpha \frac{\tau'(x)}{1 + \tau(x)} \begin{matrix} \geq \\ \leq \end{matrix} 0.$$

as the remaining part of $Y'(x)$ is strictly positive. I use the fact that $\tau'(x) = \frac{-\phi q'(x)}{(q(x) - \phi)^2}$, the definition of $\tau(x)$ and that $q(x) = \frac{1}{x} f(x)$ to get:

$$\frac{1}{x} \begin{matrix} \geq \\ \leq \end{matrix} -\alpha \phi \frac{q'(x)}{f'(x) \cdot (q(x) - \phi)}.$$

It is easy to see that $f'(x) = M_2(1, x)$ and $q'(x) = M(1, x) \frac{-1}{x^2} + \frac{1}{x} M_2(1, x)$. Using this fact yields:

$$\frac{q(x) - \phi}{\alpha \phi} \begin{matrix} \geq \\ \leq \end{matrix} \frac{M(1, x) \frac{1}{x}}{M_2(1, x)} - 1.$$

I use the CRS property of M and the fact that $q(s) = M(\frac{1}{s}, 1)$ to obtain:

$$\frac{q(x) - \phi}{\alpha \phi q(x)} + \frac{1}{q(x)} \begin{matrix} \geq \\ \leq \end{matrix} \frac{1}{f'(x)}.$$

It is easy to see that the LHS decreases in x and the RHS increases in x (by strict concavity of M). This means that if solution to $LHS = RHS$ exists then it is unique. Existence follows if we reformulate the condition above:

$$f'(x) = \frac{\alpha \phi q(x)}{q(x) - \phi(1 - \alpha)}.$$

The LHS is decreasing (and its limit is $+\infty$ at 0) and the RHS increases with x (and its limit is $+\infty$ for $x_{\alpha\phi}$ that solves $q(x_{\alpha\phi}) = \phi(1 - \alpha)$). This means that there exists $x_P \in (0, x_{\alpha\phi})$ such that $Y'(x_P) = 0$. Moreover, if α is sufficiently low then $x_P < \bar{x}$.

Convexity of function $\tau(x)$. The easiest way to show this fact is to calculate τ'' and prove that it is positive. First, notice that:

$$\tau'(x) = \frac{-\phi q'(x)}{(q(x) - \phi)^2}$$

this follows directly from the definition of $\tau(x)$. It is clear that $q'(x) < 0$ which implies that $\tau'(x) > 0$.

Second derivative reads:

$$\tau''(x) = \frac{-\phi q''(x) \cdot (q(x) - \phi) + 2\phi (q'(x))^2}{(q(x) - \phi)^3}.$$

Since $q(x) = \frac{1}{x}M(1, x)$ then:

$$q'(x) = -\frac{1}{x^2}M(1, x) + \frac{1}{x}M_2(1, x).$$

$$q''(x) = \frac{2}{x^3}M(1, x) - \frac{2}{x^2}M_2(1, x) + \frac{1}{x}M_{22}(1, x).$$

Since the denominator of $\tau''(x)$ is always positive let us focus on the numerator:

$$\begin{aligned} & -\phi q''(x) \cdot (q(x) - \phi) + 2\phi (q'(x))^2 \\ &= -\phi \left[\frac{2}{x^3}M(1, x) - \frac{2}{x^2}M_2(1, x) + \frac{1}{x}M_{22}(1, x) \right] \cdot \left(\frac{1}{x}M(1, x) - \phi \right) \\ & \quad + 2\phi \left(-\frac{1}{x^2}M(1, x) + \frac{1}{x}M_2(1, x) \right)^2 \\ &= -\frac{2\phi}{x^3}M_2(1, x)M(1, x) + \frac{2\phi^2}{x^3}M(1, x) - \frac{2\phi^2}{x^2}M_2(1, x) \\ & \quad + \frac{2\phi}{x^2}(M_2(1, x))^2 - \frac{\phi}{x}M_{22}(1, x)\{q(x) - \phi\} \\ &= M(1, x)\frac{2\phi}{x^3}(-M_2(1, x) + \phi) - M_2(1, x)\frac{2\phi}{x^2}(-M_2(1, x) + \phi) \\ & \quad - \frac{\phi}{x}M_{22}(1, x)\{q(x) - \phi\} \\ &= \frac{2\phi}{x^2}\{\phi - M_2(1, x)\} \cdot \left(M(1, x)\frac{1}{x} - M_2(1, x) \right) \\ & \quad - \frac{\phi}{x}M_{22}(1, x)\{q(x) - \phi\} \\ &= \frac{2\phi}{x^2}\{\phi - M_2(1, x)\} \cdot (q(x) - M_2(1, x)) \\ & \quad - \frac{\phi}{x}M_{22}(1, x)\{q(x) - \phi\} \\ &> \frac{2\phi}{x^2}\{\phi - M_2(1, x)\}^2 - \frac{\phi}{x}M_{22}(1, x)\{q(x) - \phi\} > 0 \end{aligned}$$

where the first inequality follows by the fact that $q(x) > \phi$ for $x \in (0, \bar{x})$ and the last inequality holds because $M_{22}(1, x) < 0$ (by the strict concavity of M).

Alternative specification of search costs. To show that the main result of my analysis (about the possibility of coexistence of a positive government multiplier and positive response of private consumption to government spending) does not depend on the specification of search costs, I analyze the model with disutility from search, as in [Bai et al. (2011)]. Let us start with the model that is

analogous to the one presented in Section 3.3. The problem that is solved by households reads:

$$\max_{c,m,v} \log c + \chi \log m - G(v)$$

subject to :

$$c = q(x)v$$

$$pc + m = \mu + f(x) \cdot 1$$

where the notation is the same as in the core text and G is a function that describes disutility from making visits. In particular, it is assumed that G is linear, i.e.:

$$G(v) = \chi_v v$$

and $\chi_v > 0$. Observe, that households are producers of goods and hence there are no firms in this version of the model. The reason for this reformulation is discussed later. I solve the household's maximization problem in a similar way to the one presented in Section 3.3 and I obtain the following FOC:

$$c(x) = \frac{1}{\frac{\chi p}{\mu} + \frac{\chi_v}{q(x)}}$$

that describes the consumer's demand for goods (recall that p is a parameter and hence the demand is a function that depends solely on x). Observe that $c'(x) < 0$. The resource constraint (and at the same time the equilibrium condition) for this economy is:

$$c(x) = f(x) \cdot 1.$$

Since $f'(x) > 0$, $f(0) = 0$, $f(x) > 0$ for $x > 0$ and since $q(0) > 0$, $\lim_{x \rightarrow +\infty} q(x) = 0$ then the equation above has a unique solution. It is easy to show that government intervention (in this case government does not bear any search costs as it is hard to define the concept of government's search disutility) characterized by the purchase of $g > 0$ goods leads to the following modification of the resource constraint:

$$c(x) + g = f(x) \cdot 1$$

and a simple use of the Implicit Function Theorem implies that $x'(g) > 0$ which implies that private consumption drops and output $f(x) \cdot 1$ increases.

Let us turn to a model with search frictions where agents purchase goods from each other to generate their own output. In what follows I consider households that not only consume but also are able to produce goods. This formulation is motivated by the fact that considering a situation in which firms and households are separate entities and the former have to make visits to buy inputs implies that one has to define firm's disutility from search activities so that it is symmetric to consumer's search process. To avoid this methodological problem I assume WLOG that households are producers

at the same time. This means that consumer's-producer's problem reads:

$$\max_{c, m, v_s, v_f, y} \log c + \chi \log m - G(v_s) - G(v_f)$$

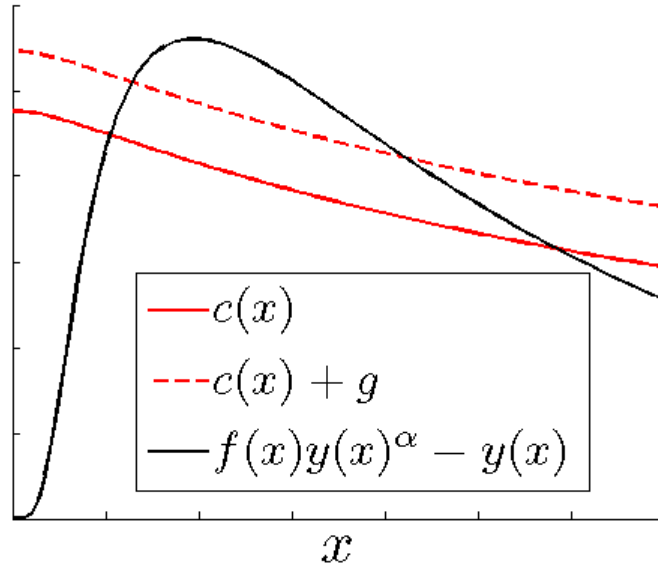
$$c = q(x)v_s$$

$$y = q(x)v_f$$

$$pc + m = \mu + pf(x)y^\alpha - py$$

where v_s is the number of visits made by households to get consumption goods and v_f are visits made to get inputs for the household's "factory". Moreover, it is assumed that $G(v) = \chi_v \cdot v$ with $\chi_v > 0$. There are two FOCs that describe household's solution and implicitly define the demand side and the supply side of economy:

Figure 22: Two equilibria in the model with search disutility



$$c(x) = \frac{1}{\frac{\chi p}{\mu} + \frac{\chi v}{q(x)}}$$

$$y(x) = \left(\frac{\alpha f(x)}{1 + \frac{\chi v \mu}{p \chi q(x)}} \right)^{\frac{1}{1-\alpha}}.$$

The resource constraint combined with $c(x)$ and $y(x)$ yields:

$$c(x) = f(x)y(x)^\alpha - y(x).$$

Since the analytic argument that shows the existence of two solutions in equation above is hard to formulate, I rely on numerical simulation, instead. Figure 22 shows that the equilibrium condition has two solutions and government intervention.⁷³ This shows that the main result of this work is independent of specification of search costs.

⁷³I have chosen the following parameter values for the simulation: $\mu = 0.45$, $\chi = 1$, $\chi_v = 1$, $p = 9$, $L = 2$ (parameter associated with the Den Haan - Ramey - Watson specification of the matching function), $\alpha = 0.8$.

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